HIGHLY ORDER COMPACT FINITE DIFFERENCE METHODS COMBINED WITH SMOOTHING SCHEME FOR SOLVING KURAMOTO- SIVASHINSKY EQUATION

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In this paper, we develop a numerical solution for the well-known KuramotoSivashinsky equation. The equation is a fourth-order nonlinear partial differential equation with a non-negative coefficient parameter. We use a highly order compact finite difference method for spatial discretization,and then apply smoothing ETD (1,3)-Pade scheme in time. We will also study the effect of the coefficient parameter of the nonlinear term on the behavior of the numerical solution. The numerical results and the figures show good agreement with the expected asymptotic behavior of the coefficient parameter.

Keywords: Kuramoto- Sivashinsky equation ,Compact finite difference, ETD(1,3)-Pade scheme

1. **Introduction**

There has been considerable attention to the K-S equation in recent years (see for example, [1,2,3,4,5,6,7,8]).The K-S equation was originally derived by Kuramoto in context of angular-phase turbulence, for a system of reaction-diffusion equations modeling the Belouzov- Zabotinskii reaction in three space dimensions [9,10]; he considered $u(x_1, x_2, x_3, t)$ to be a small perturbation of a global periodic solution , just beyond the parameter domain, where the Hopf bifurcation has occurred . The K- S equation was also derived independently by Sivashinsky , while modelling small thermal diffusive instabilities in laminar flame fronts [11,12,13]. In this case, $u(x_1, x_2, x_3, t)$ is the perturbation of an unstable planar flame front in the direction of propagation ; therefore, both the work of Kuramoto and Sivashinsky were motivated by the study of nonlinear stability of traveling waves. Hence, the qualitative and quantitative study of the K-S equation is of great interest, analogizing the Burgers and Navier - Stocks equations. The K-S equation has been used also in pattern formation modeling and other areas of applications. A list of references related to these applications, is given in [14]. In addition to the physical motivations, the K-S equation because of its rich dynamical properties is of significant mathematical interest [15, 16]. Of particular significance is the connection with the theory of inertial manifolds. When used as a model or when seen as a mathematically relevant dynamical system ,the K-S equation requires numerical solutions. Various numerical methods have been introduced by researchers to solve K-S equation. See [17,18,19,20] for more information in that regard. In the current study , it is aimed to employ the high-order compact finite difference methods for discretizing spatial derivatives and modified Exponential Time Differencing (ETD)(1,3)-Pade scheme[21,22] in time. The

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compact method needs three nodes to obtain a fourth-order which has less truncation error in comparison to the conventional finite difference schemes .Various versions of the compact finite difference schemes were analyzed and implemented successfully by the same researchers to deal with their own problems[23,24] and references therein. The ETD(1,3)-Pade scheme is based on Exponential Time Differencing Runge-Kutta fourth-order (ETDRK4)with sub-diagonal Pade scheme to matrix exponential functions that offers an algorithm in parallel forms . Combination of the high- order compact finite difference in space with ETD(1,3)-Pade scheme provides accurate solutions for the K-S equation. This method does not require specific transformations for nonlinear terms as required by some existing techniques. We consider the following one -dimensional K-S equation:

$$u_t + \mu u u_x + \mu u_{xx} + 4u_{xxxx} = 0 \qquad 0 \le x \le b, \quad 0 \le t \le T \tag{1}$$

Subject to the periodic boundary condition:

$$u(x+b,t) = u(x,t) \quad 0 \le t \le T \tag{2}$$

with initial condition:

$$u(x,0) = f(x) \quad 0 \le x \le b \tag{3}$$

where μ is a nonnegative parameter and $f(x)$ is a known function. Eq(1) contains both second and fourth-order derivatives; therefore, it produces complex behavior. The secondorder term acts as an energy source and has a destabilizing effect, and the nonlinear term transfers energy from low to high wave numbers where the fourth-order term has a stabilizing effect.It is a PDE that can exhibit chaotic solutions.

The paper is organized as follows: In section 2 we describe the compact finite difference method. The smoothing scheme is presented in section3. In section 4 a numerical example is provided and section5 deals with the conclusions.

Discretization

We begin first by subdividing the range $a \le x \le b$ into N subintervals of width $h = (b-a)/N$. We use x_i to denote the points of subdivision, where $x_i = a + ih$ for $i = 0, 1, 2, ..., N$, so that $x_0 = a, x_N = b$.

Next,we subdivide the time range $0 \le t \le T$ into M subintervals.In the temporal dimension, the uniform increment $\triangle t$ is used;thus,$t_n = n\triangle t$ is the time level for nth step. The quantity u_i^n represents the numerical solution at (x_i, t_n).

2. **Compact finite difference method**

The usual objection to fourth and sixth-order schemes comes from the additional nodes necessary to achieve the higher order accuracy. Besides the attendant difficulties of having to consider two fictitious nodes when a boundary point is being computed, the additional nodes almost preclude the use of fourth and sixth-order implicit methods since the matrix which arises is not the simple tridiagonal form produced by secondorder schemes. We propose fourth and sixth-order tridiagonal forms.The first, second, and fourthorder derivative values are evaluated by the compact difference formulae. Much works has been done in deriving such formulae.We quote the formulae in Ref [21].We omit the time level symbol for convenience.

2.1. Fourth- order compact finite difference method(FOCM)

In [21],values of a function on a set of nodes of the finite difference approximation to the derivative of the function is expressed as a linear combination of the given function values.

For the firstorder derivative, the generalization form is

$$\beta u'_{i-2}+\alpha u'_{i-1}+u'_i+\alpha u'_{i+1}+\beta u'_{i+2}=c\frac{u_{i+3}-u_{i-3}}{6h}+b\frac{u_{i+2}-u_{i-2}}{4h}+a\frac{u_{i+1}-u_{i-1}}{2h} \quad (4)$$

for $\beta=0, a=2/3(\alpha+2), b=1/3(4\alpha-1), c=0$ in (4) we have :

$$\alpha u'_{i-1}+u'_i+\alpha u'_{i+1}=\frac{\alpha}{3h}(u_{i+2}+u_{i+1}-u_{i-1}-u_{i-2})+\frac{1}{12h}(8u_{i+1}-8u_{i-1}-u_{i+2}+u_{i-2}) \quad (5)$$

by setting $\alpha=1/4$ in (5), we arrive to the fourth-order central difference scheme.

$$u'_{i-1}+4u'_i+u'_{i+1}=\frac{3}{4h}(u_{i+1}-u_{i-1}) \qquad i=2,...,N-1 \quad (6)$$

where $u'_i=\frac{du_{x_i}}{dx}$,and the truncation error for Eq(6)is fourth order. The matrix form with considering periodic boundary condition is

$$M_1=\begin{bmatrix} 4 & 1 & 0 & \dots & 1 \\ a1 & 4 & 1 & \dots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \dots & 1 & 4 & 1 \\ 1 & \dots & 0 & 1 & 4 \end{bmatrix}, A_1=\begin{bmatrix} 0 & 1 & 0 & \dots & -1 \\ -1 & 0 & 1 & \dots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \dots & -1 & 0 & 1 \\ 1 & \dots & 0 & -1 & 0 \end{bmatrix}$$

$$U=\begin{bmatrix} u_1 \\ \vdots \\ u_n \end{bmatrix}, U'=\begin{bmatrix} u'_1 \\ \vdots \\ u'_n \end{bmatrix}, U'=M_1^{-1}A_1U \quad (7)$$

For secondorder derivatives,

$$\beta u''_{i-2}+\alpha u''_{i-1}+u''_i+\alpha u''_{i+1}+\beta u''_{i+2}= c\frac{u_{i+3}-2u_i+u_{i-3}}{9h^2}+b\frac{u_{i+2}-2u_i+u_{i-2}}{4h^2}+a\frac{u_{i+1}-2u_i+u_{i-1}}{h^2} \quad (8)$$

for,$\beta=0, a=4/3(1-\alpha), b=1/3(-1+10\alpha), c=0$

$$\alpha u_{i+1}+u_i+\alpha u_{i-1}=\frac{10\alpha-1}{12h^2}(u_{i+2}-2u_i+u_{i-2})\frac{4(1-\alpha)}{3h^2}(u_{i+1}-2u_i+u_{i-1}) \quad (9)$$

by setting $\alpha=1/10$ in (9) we have

$$u_{i-1}+10u_i+u_{i-1}=\frac{12}{h^2}(u_{i+2}-2u_i+u_{i-2}) \quad i=1,2,...,N-1 \quad (10)$$

$$M_2=\begin{bmatrix} 10 & 1 & 0 & \dots & 1 \\ 1 & 10 & 1 & \dots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \dots & 1 & 10 & 1 \\ 1 & \dots & 0 & 1 & 10 \end{bmatrix}, A_2=\frac{12}{h^2}\begin{bmatrix} -2 & 1 & 0 & \dots & 1 \\ 1 & -2 & 1 & \dots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \dots & 1 & -2 & 1 \\ 1 & \dots & 0 & 1 & -2 \end{bmatrix}$$

where

$$U^{''}=\begin{bmatrix} u_1^{''} \\ \vdots \\ u_n^{''} \end{bmatrix}, U^{''}=M_2^{-1}A_2U \tag{11}$$

And for the fourth-order derivatives formulae:

$$\alpha u_{i+1}^{(4)}+u_i^{(4)}+\alpha u_{i-1}^{(4)}=a\frac{u_{i+2}-4u_{i+1}+6u_i-4u_{i-1}+u_{i-2}}{h^4} \tag{12}$$

for $a=2(1-\alpha), b=4\alpha-1$ in (12),

$$\begin{aligned}\alpha u_{i+1}^{(4)}+u_i^{(4)}+\alpha u_{i-1}^{(4)}=& \\ &\frac{4\alpha-1}{6h^4}(u_{i+3}-9u_{i+1}+16u_i-9u_{i-1}+u_{i-3}) \\ &+\frac{2(1-\alpha)}{h^4}(u_{i+2}-4u_{i+1}+6u_i-4u_{i-1}+u_{i-2})\end{aligned} \tag{13}$$

by setting $\alpha=1/4$ in (13),we have

$$u_{i+1}^{(4)}+4u_i^{(4)}+u_{i-1}^{(4)}=\frac{6}{h^4}(u_{i+2}-4u_{i+1}+6u_i-4u_{i-1}+u_{i-2}) \tag{14}$$

Then, the matrix forms is

$$M_4=\begin{bmatrix} 4 & 1 & 0 & \dots & 1 \\ 1 & 4 & 1 & \dots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \dots & 1 & 4 & 1 \\ 1 & \dots & 0 & 1 & 4 \end{bmatrix}, A_4=\begin{bmatrix} 6 & -4 & 1 & 0 & \dots & 1 & -4 \\ -4 & 6 & -4 & 1 & \dots & 0 & 1 \\ 1 & -4 & 6 & -4 & 1 & \dots & \vdots \\ 0 & \ddots & \ddots & \ddots & \dots & \dots & \vdots \\ \vdots & \dots & 1 & -4 & 6 & -4 & 1 \\ 1 & \dots & 0 & 1 & -4 & 6 & -4 \\ -4 & 1 & \dots & 0 & 1 & -4 & 6 \end{bmatrix}$$

where

$$U^{(4)}=\begin{bmatrix} u_1^{(4)} \\ \vdots \\ u_n^{(4)} \end{bmatrix}, U^{(4)}=M_4^{-1}A_4U \tag{15}$$

3. **Sixth order compact finite difference method (SOCM)**

Similarly, for $\alpha=1/3$ in (5), $\alpha=2/11$ in (9) and $\alpha=7/26$ in (13), the leading order truncation error coefficient vanishes, and the scheme is formally sixth-order accurate.

$$u_{i-1}^{'}+3u_i^{'}+u_{i+1}^{'}=\frac{1}{12h}(u_{i+2}-u_{i+2})+\frac{7}{3h}(u_{i+1}-u_{i-1}) \qquad i=1,...,N-1 \tag{16}$$

$$u_{i-1}^{''}+(11/2)u_i^{''}+u_{i+1}^{''}=\frac{3}{8h^2}(u_{i+2}-2u_i+u_{i-2})+\frac{6}{h^2}(u_{i+1}-2u_i+u_{i-1}) \quad i=1,...,N-1 \tag{17}$$

$$u_{i-1}^{(4)} + (26/7)u_i^{(4)} + u_{i-1}^{(4)} = \frac{1}{21h^2}(u_{i+3} - 9u_{i+1} + 16u_i - 9u_{i-1} + u_{i-3}) + \frac{38}{7h^4}(u_{i+2} - 4u_{i+1} + 6u_i - 4u_{i-1} + u_{i-2}) \quad (18)$$

Then we have the following matrix forms

$$M_1 = \begin{bmatrix} 3 & 1 & 0 & \dots & 1 \\ 1 & 3 & 1 & \dots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \dots & 1 & 3 & 1 \\ 1 & \dots & 0 & 1 & 3 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} 0 & 7/3 & 1/12 & 0 & \dots & -1/12 & -7/3 \\ -7/3 & 0 & 7/3 & 1/12 & \dots & 0 & -1/12 \\ -1/12 & -7/3 & 0 & 7/3 & 1/12 & \dots & \vdots \\ 0 & \ddots & \ddots & \ddots & \dots & \dots & \vdots \\ \vdots & \dots & -1/12 & -7/3 & 0 & 7/3 & 1/12 \\ 1/12 & \dots & 0 & -1/12 & -7/3 & 0 & 7/3 \\ 7/3 & 1/12 & \dots & 0 & -1/12 & -7/3 & 0 \end{bmatrix}$$

$$M_2 = \begin{bmatrix} 11/2 & 1 & 0 & \dots & 1 \\ 1 & 11/2 & 1 & \dots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \dots & 1 & 11/2 & 1 \\ 1 & \dots & 0 & 1 & 11/2 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -51/4 & 6 & 3/8 & 0 & \dots & 3/8 & 6 \\ 6 & -51/4 & 6 & 3/8 & \dots & 0 & 3/8 \\ 3/8 & 6 & -51/4 & 6 & 3/8 & \dots & \vdots \\ 0 & \ddots & \ddots & \ddots & \dots & \dots & \vdots \\ \vdots & \dots & 3/8 & 6 & -51/4 & 6 & 3/8 \\ 3/8 & \dots & 0 & 3/8 & 6 & -51/4 & 6 \\ 6 & 3/8 & \dots & 0 & 3/8 & 6 & -51/4 \end{bmatrix}$$

and

$$M_4 = \begin{bmatrix} 26/7 & 1 & 0 & \dots & 1 \\ 1 & 26/7 & 1 & \dots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \dots & 1 & 26/7 & 1 \\ 1 & \dots & 0 & 1 & 26/7 \end{bmatrix}$$

$$A_4 =$$

$$\begin{bmatrix} 100/3 & -155/7 & 38/7 & 1/21 & 0 & \dots & 0 & 1/21 & 38/7 & -155/7 \\ -155/7 & 100/3 & -155/7 & 38/7 & 1/21 & 0 & \dots & 0 & 1/21 & 38/7 \\ 38/7 & -155/7 & 100/3 & -155/7 & 38/7 & 1/21 & 0 & \dots & 0 & 1/21 \\ 1/21 & 38/7 & -155/7 & 100/3 & -155/7 & 38/7 & 1/21 & 0 & \dots & 0 \\ \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\ \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\ 1/21 & 0 & \dots & 0 & 1/21 & 38/7 & -155/7 & 100/3 & -155/7 & 38/7 \\ 38/7 & 1/21 & 0 & \dots & 0 & 1/21 & 38/7 & -155/7 & 100/3 & -155/7 \\ -155/7 & 38/7 & 1/21 & 0 & \dots & 0 & 1/21 & 38/7 & -155/7 & 100/3 \end{bmatrix}$$

4. Smoothing scheme for solving nonlinear ODEs

By replacing(7),(11),(15)in equation (1)with considering periodic boundary conditions, we get the system of ODEs

$$\frac{du_i}{dt} = -\mu u_i M_1^{-1} A_1 u_i - \mu M_2^{-1} A_2 u_i - 4M_4^{-1} A_4 u_i \quad i = 1, ..., N \tag{19}$$

Briefly,

$$U_t + AU = F(u, t) \tag{20}$$

where$A = \mu M_2^{-1} A_2 u_i + 4M_4^{-1} A_4 u_i$ is a matrix representing a spatial discretization of linear terms and $F(u, t) = -\mu u_i M_1^{-1} A_1 u_i$ is the discretization vector of the nonlinear term.
Let $k = t_{n+1} - t_n$ be the time step size, then using a variation of constant formulae, we come up with the following recurrence formulae :

$$u(t_{n+1}) = e^{-kA} u(t_n) + k \int_0^1 e^{-kA(1-\tau)} F(u(k + \tau k), t_n + \tau k) d\tau \tag{21}$$

The expression (21) is an exact solution to (20) and is the basis of different time-stepping schemes, depending on how one approximates the matrix exponential functions and the integral term.The ETD (exponential time differencing) schemes solve the linear part exactly and then explicitly approximates the integral part by polynomial approximation. Cox and Matthews [19] developed a class of Exponential Time Differencing Runge-Kutta type schemes for nonlinear stiff systems.These schemes suffer from computational difficulties because matrix inverses and matrix exponential functions are needed to compute. In[1] Khaliq and Vaquero introduced a modification of the (ETDRK4)Cox and Matthews scheme. This scheme not only solve the problem of numerical instability but also computational difficulties. They utilized diagonal Pade schemes to approximate matrix exponential functions. To deal with the problem of non-smooth data, they combined diagonal Pade schemes with the positivity- preserving sub-diagonal schemes and used partial fraction decomposition of the rational matrix functions, which makes their scheme efficient and allows one instead to solve several well-conditioned linear problems. Unfortunately, they were unable to provide a threshold value for the number of initial steps of the smoothing scheme required to damp spurious oscillations due to the non-smooth data. In [13] Khaliq, et al. proposed a modification of [1]and introduced the most efficient and stable algorithm without the problem of threshold value.

3.1. The ETD(1,3)-Pade scheme parallel algorithm

In order to implement scheme ETD (1,3)-Pade scheme in a computationally efficient manner, we present a description of the algorithm by implementing a partial fraction splitting technique. See[1,13]for more information on this regard.
for $i = 1, ..., q_1 + q_2$
step1. Solve

$$(kA - \tilde{c}_i I) R_{a_i} = \tilde{w}_i U_n + k\Omega_i F(u_n, t_n),$$

For R_{a_i} compute a_n as:

$$a_n = \sum_{i=1}^{q_1} R_{a_i} + 2 \sum_{1+q_1}^{q_1+q_2} Re(R_{a_i}).$$

step2. Solve

$$(kA - \tilde{c_i}I)R_{b_i} = \tilde{w_i}U_n + k\Omega_i F(a_n, t_n + k/2),$$

For R_{b_i} compute b_n as:

$$b_n = \sum_{i=1}^{q_1} R_{b_i} + 2 \sum_{1+q_1}^{q_1+q_2} Re(R_{b_i}).$$

step3 .Solve

$$(kA - \tilde{c_i}I)R_{c_i} = \tilde{w_i}a_n + k\Omega_i(2F(b_n, t_n + k/2) - F(U_n, t_n),$$

For R_{c_i} compute c_n as:

$$c_n = \sum_{i=1}^{q_1} R_{c_i} + 2 \sum_{1+q_1}^{q_1+q_2} Re(R_{c_i}).$$

step4 .Solve

$$(kA - \tilde{c_i}I)R_{u_i} = \tilde{w_i}u_n + k\omega_{1i}(2F(U_n, t_n) + k\omega_{2i}(F(a_n, t_n + k/2) + F(b_n, t_n + k/2))$$
$$+k\omega_{3i}F(c_n, t_n + k)$$

For R_{u_i} compute u_{n+1} as:

$$U_{n+1} = \sum_{i=1}^{q_1} R_{u_i} + 2 \sum_{1+q_1}^{q_1+q_2} Re(R_{u_i}).$$

The poles and corresponding weights are as follows:

$$c_1 = -2.6258168189584667160,$$
$$c_2 = -1.6870915905207666420 - i2.5087317549248805108,$$
$$\omega_1 = 5.5407990186788211678,$$
$$\omega_2 = -2.7703995093394105839 - i0.1591864442851235025,$$
$$\omega_{11} = 0.92346650311313686128,$$
$$\omega_{12} = -0.46173325155656843064 - i0.02653107404752058375 0,$$
$$\omega_{21} = 0.38305077592917562056,$$
$$\omega_{22} = -0.19152538796458781028 + i0.47027336073401897817,$$
$$\omega_{31} = 0.42055591813817669094,$$
$$\omega_{32} = 0.28972204093091165453 - i0.18298527878713726274,$$
$$\tilde{c_1} = -5.2516336379169334320,$$
$$\tilde{c_2} = -3.3741831810415332840 - i5.0174635098497610217,$$
$$\tilde{\omega_1} = 11.081598037357642334,$$
$$\tilde{\omega_2} = -5.5407990186788211672 - i0.31837288857024700490,$$
$$\tilde{\Omega_1} = 2.1101239731096647932,$$
$$\tilde{\Omega_2} = -0.55506198655483239660 + i0.73103036863338010983.$$

The ETD(1,3)-Pade scheme contains 4 stages and requires the solution of eight backward Euler type linear systems at each time step. Each of these stages contain two sub-steps and they are independent of each other. This design of ETD(1,3)-Pade scheme offers parallel implementation of the algorithm on a two processor computer.

Linear stability analysis
The linear stability of the ETD(1,3)-Pade scheme for nonlinear ODE investigated in [13] by adopting an approach is discussed in [11,20,19].

5. **Numerical example**

We seek the numerical solution for the following problem for $\mu = 3\&1$,

$$u_t + \mu u u_x + \mu u_{xx} + 4u_{xxxx} = 0 \qquad 0 \leq x \leq 2\pi, \quad 0 \leq t \leq T \tag{22}$$

subject to the periodic boundary condition

$$u(x + 2\pi, t) = u(x, t) \quad 0 \leq t \leq T \tag{23}$$

with initial condition

$$u(x, 0) = \cos(x) \quad 0 \leq x \leq 2\pi \tag{24}$$

We carried out various numerical experiments to investigate the behavior of the solution of the K-S equation. We solved the K-S equation by second-order compact finite difference method using 1000 point. Considered as an exact solution, and compare with solutions obtained using the fourth and sixth order compact methods (FOCM and SOCM), for these two methods, we took 100 points.

$u(x,t)$	$x = 0$	$x = \pi/5$	$x = \pi/2$	$x = \pi$	$x = 3\pi/2$	$x = 2\pi$
FOCM	0.40904598	0.33852930	0.01548713	−0.38934408	−0.00752176	0.40770864
SOCM	0.40904600	0.33852932	0.01548713	−0.38934410	−0.00752176	0.40770866

Table 1. Values of u by using FOCM and SOCM methods with ETD(1,3)scheme for =3,at t=1s.

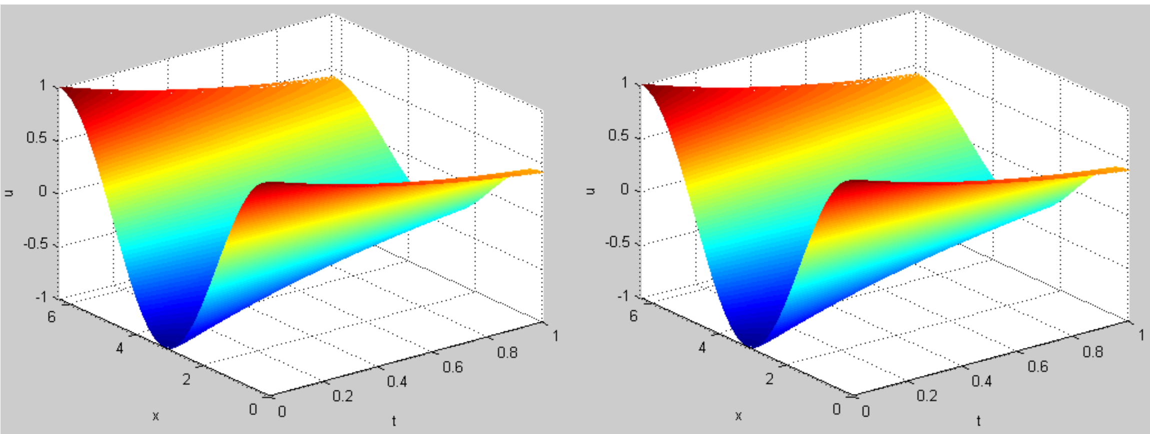

Figure 1. Values of K-S equation by FOCM (Left)and SOCM(Right) methods with ETD(1,3)-Pade scheme, for $\mu = 3$, at $t = 1$s.

$u(x,t)$	$x = 0$	$x = \pi/5$	$x = \pi/2$	$x = \pi$	$x = 3\pi/2$	$x = 2\pi$
FOCM	0.06718259	0.055767246	0.00956788	−0.04739023	0.00845336	0.06714397
SOCM	0.06718259	0.05576724	0.00956788	−0.04739023	0.00845336	0.06714397

Table 2. Values of u by using FOCM and SOCM methods with ETD (1,3) Pade scheme, for =1, at t=1s.

Table1 and 2 shows the values of$u$ obtained by FOCM and SOCM methods for $\mu = 3, 1$,for $x \in [0, \pi]$ at $t = 1$s. The values of errors are presented in table3 and 4. They have the fourth-order accuracy. As we see , the errors of SOCM method with ETD(1,3)-Pade scheme are almost the same as FOCM method with ETD(1,3)-Pade scheme. Therefore , the FOCM method is better than SOCM because of less computational cost . We look at the behavior solution of various values of μ . Figure 3 shows the solutions of the K-S equation by using FOCM at$t = 1$s for parameter values $\mu = 0, 3, 6, 8, 15$ and 20. In the range $0 : 8$,the maximum amplitude of the solution increases. We confirmed that the maximum amplitude of the solution is highly dependent on. Either, for μ values exceeding 15, the maximum amplitude continuously increases with increasing μ, while for smaller values the maximum amplitude decreases with increasing values of μ .The study showed the FOCM method with ETD(1,3)-Pade scheme give the best results both in terms of accuracy and efficiency as we found in the others.

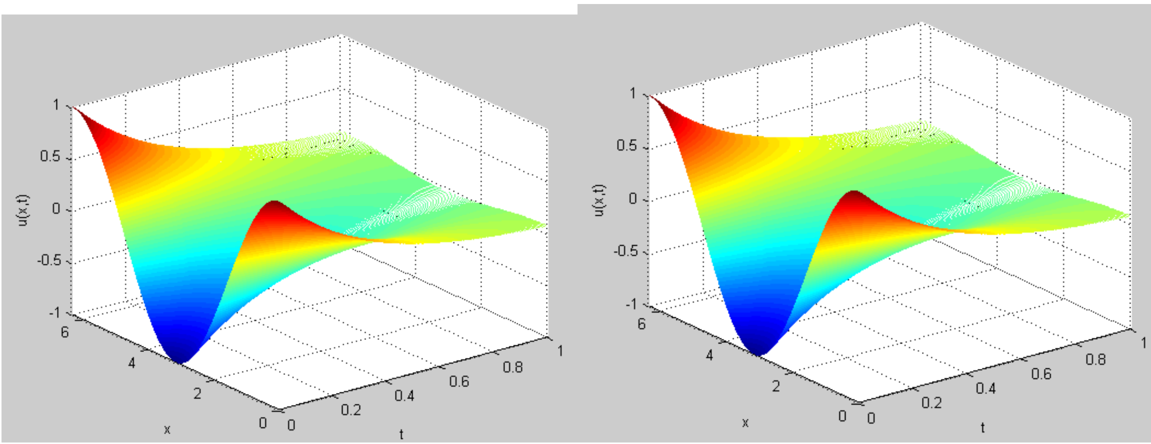

FIGURE 2. Values of K-S equation by FOCM (Left)and SOCM(Right) methods with ETD(1,3)-Pade scheme, for $\mu = 1$, at $t = 1$s.

Error	$x = 0$	$x = \pi/5$	$x = \pi/2$	$x = \pi$	$x = 3\pi/2$	$x = 2\pi$
FOCM	6.209×10^{-4}	5.251×10^{-4}	1.584×10^{-4}	6.219×10^{-4}	1.820×10^{-4}	6.169×10^{-4}
SOCM	6.209×10^{-4}	5.251×10^{-4}	1.582×10^{-6}	6.219×10^{-4}	1.820×10^{-5}	6.169×10^{-4}

TABLE 3. Error of FOCM ,SOCM methods with ETD(1,3)-pade scheme for $\mu = 3$,at $t = 1$s

FOCM	1.240×10^{-4}	9.927×10^{-5}	1.080×10^{-6}	1.241×10^{-4}	2.772×10^{-6}	1.240×10^{-4}
SOCM	1.240×10^{-4}	9.927×10^{-5}	1.080×10^{-6}	1.241×10^{-4}	2.772×10^{-6}	1.240×10^{-4}

TABLE 4. Error of FOCM ,SOCM methods with ETD(1,3)-pade scheme for $\mu = 1$,at $t = 1$s

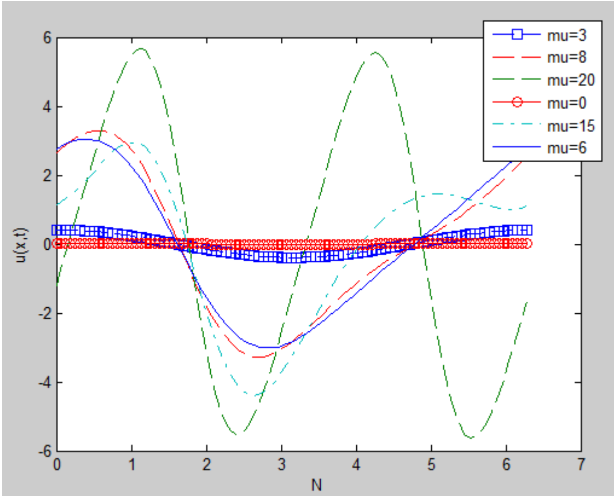

FIGURE 3. The solutions of K-S equation for $\mu = 0, 3, 6, 8, 15$ and 20 at $t = 1s$.

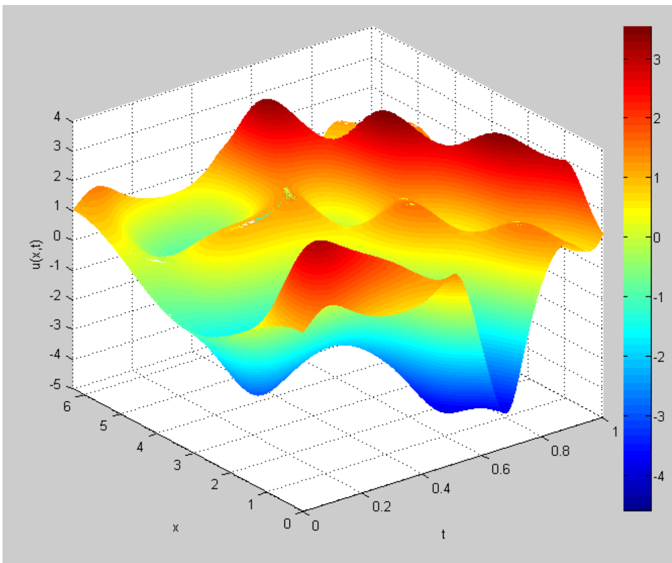

FIGURE 4. The values of K-S equation by FOCM with ETD (1,3)-Pade scheme,for $\mu = 15$, at t=1s.

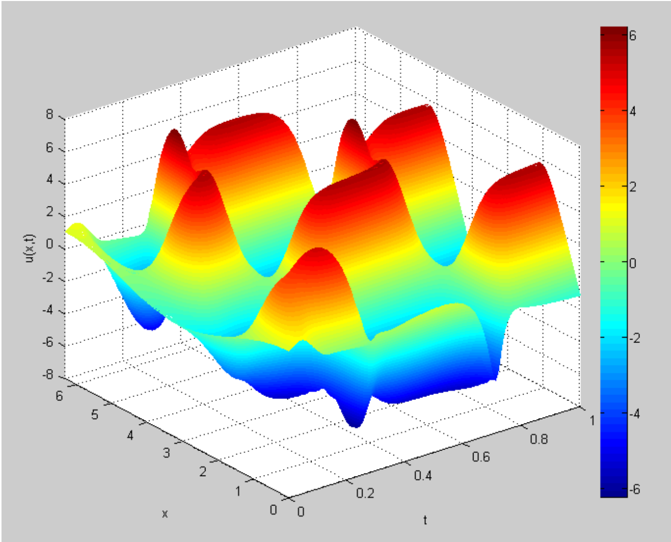

FIGURE 5. The values of K-S equation by FOCM with ETD(1,3)-Pade scheme,for $\mu = 20$,at t=1s.

6. Conclusions

In this article, two high-order compact finite difference methods have been developed for the numerical solution of Kuramoto- Sivashinsky equation . By using these methods the problem is reduced to a system of ODEs which are solved by ETD(1,3)- Pade scheme .We investigated the performance of compact schemes including fourth and sixth-order methods in order to discretization of spatial derivatives , and for ODEs system , we have used from the ETD(1,3)-Pade scheme in the form of partial fraction splitting technique .This method solved the difficulties of high-order methods and nonsmooth data . Moreover , the method work with good accuracy and efficiency ,the given numerical example support this claim.

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