CONVOLUTIONAL /TURBO ENCODING IMPROVEMENTS FOR DIFFERENT MULTIUSER DETECTION ALGORITHMS IN UNPERFECT RECEPTION CONDITIONS

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În ultimii ani s-a observat o extindere a modalităților de implementare, a capacităților şi performanțelor sistemelor multiutilizator. Structurile complexe necesită atenție deosebită atunci când trebuie selectați cei mai potriviți parametrii pentru aceste sisteme. În sistemul de tip CDMA (Code Division Multiple Access) un rol determinant în vederea obținerii unor performane deosebite îl au puterea semnalelor prin detectoare, tehnicile de modulație, de codare/decodare utilizate, precum şi alegerea corectă a secvențelor de împrăştiere.

Lucrarea are drept scop analiza performanțelor detectoarelor multiutilizator de tip convențional, optimal şi MMSE (Minimum Mean Square Error) în condiții de lucru variabile. În capitolul final vor fi subliniate principalele concluzii legate de cel mai performant sistem, analiza realizându-se pentru semnale ale utilizatorilor cu puteri diferite şi variate secvențe de împrăştiere. În plus, pe lângă variația secvențelor de împrăştiere, sunt analizate şi efectele utilizării tehnicilor de codare/decodare convoluțională şi turbo.

Mediul de simulare este Matlab, iar rezultatele obținute pentru cele 3 tipuri de detectoare vor fi ilustrate în ultimul capitol ca figuri ce reprezintă variațiile BER (Rata de Eroare de Bit) în funcție de RSZ (Raportul Semnal-Zgomot). Pe baza rezultatelor obținute se vor enumera o serie de concluzii relevante pentru analiza şi utilizarea ulterioară a acestor detectoare la nivelul stațiilor de bază.

In the latest years multiuser detection systems have supported an increase in design, capacity and performances. Their complex structure requires attention in the process of choosing the appropriate parameters. In CDMA (Code Division Multiple Access) system very important roles in achieving significant performances are played by the power of the signals that cross these detectors, coding/decoding modulation techniques used and also on the correct choice of the spreading sequences.

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Our paper aims to analyze the performances of conventional, optimum and MMSE (Minimum Mean Square Error) multiuser detectors in various conditions. Conclusions about the best performing of these systems in cases of different powers of the users signals, different types of spreading sequences will be presented in the final chapter. Our analysis includes convolutional and turbo techniques in addition to various spreading sequences.

The simulation environment used is Matlab and the results for all these detectors will be illustrated in the last chapter as figures of BER (Bit Error Rate) depending on the SNR (Signal to Noise Ratio). Based on the obtained results many relevant conclusions for the usage of these detectors at the base station level can be drawn.

Keywords: MUD (multiuser detectors), BER performances, turbo technique

1. Introduction

Multiuser detection refers to the process of demodulating one or more user data streams from a non-orthogonal multiplex. A number of approaches of this problem have been studied over the past decade. Such methods have been shown to offer very attractive performance characteristics, although this performance comes at the expense of complexity that is exponential in the number of users [1].

In multiuser systems the main distorsion that affects the performances is the multiple access interference. It is important to choose the best performing multiuser detector and also the selection of spreading sequences to differentiate the users plays an important role in the system capacity [2]. There are significant conclusions when the users are not perfectly orthogonal and/or when they have un-equal amplitude [3]. Also spreading and coding/decoding techniques have been used to increase the system performances [4], [5].

Our paper studies the behavior of conventional, optimum and MMSE detectors in various conditions. The specialty literature illustrates the general performances of such detectors that can be obtained for N users of these systems. For CDMA system there have been tested orthogonal and non-orthogonal spreading sequences, different coding/decoding techniques. Their effect over the global performances of the systems will be given in the "Simulation Results" chapter.

2. CDMA System Model

The synchronous DS-CDMA system is assumed to be used by N users each transmitting signals on a bit period. The noise on the channel is considered AWGN [6]. Each user signal has associated a corresponding spreading sequences $s_k(t)$ therefore the received signal can be written as:

$$y(t)=\sum_{k=1}^{N} A_k b_k s_k(t)+\sigma n(t), \quad t\in[0,T] \tag{1}$$

where T is bit period; $s_k(t)$ is the spreading sequence for every k user, normalized so its energy to be equal to unit; $b_k \in \{-1,1\}$ represent the bit transmitted by the kth user in T; A_k is the received amplitude from user k; $n(t)$ is the AWGN with power spectral density equal to unit and σ is the variance of the noise.

The performances of every detector depend on the SNR and resembling between user's signals, resembling reflected by the cross-correlation coefficients ρ_{ij}. The cross-correlation matrix is defined as:

$$R=\begin{pmatrix} \rho_{11} & \rho_{12} & .. & \rho_{1N} \\ \rho_{21} & \rho_{22} & .. & \rho_{2N} \\ .. & .. & .. & .. \\ \rho_{N1} & \rho_{N2} & .. & \rho_{NN} \end{pmatrix} \tag{2}$$

3. Turbo Coding/Decoding

The Turbo Encoder structure (given in fig. 1) [7] consists of two recursive Systematic Codes (RSC) that operates on the same input bits. The first encoder (RSC1) uses the systematic encoder polynomial $g_1(D)$ and the second encoder (RSC2) a nonsystematic polynomial $g_2(D)$. For the second encoder the input bits order is changed by placing an interleaver in front of it.

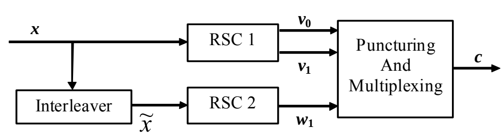

Fig. 1. The Turbo Encoder

The iterative decoder structure (fig. 2) consists of two component decoders, serially concatenated via an interleaver identical to the one used in the encoder.

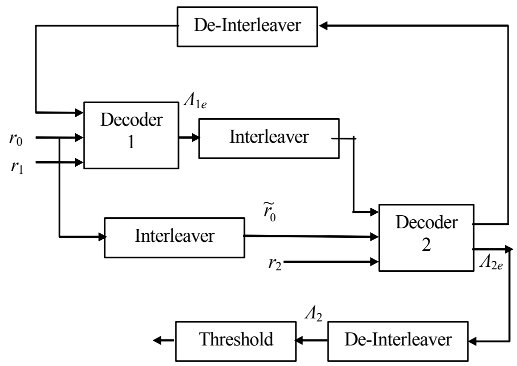

Fig. 2. The Turbo Decoder

Each decoder maximizes the Log-Likelihood ratio based on the received information bits, the parity check bits and the soft output information from the other decoder. The log-likelihood ratio can be determined using MAP, log-MAP, Max-Log-MAP and SOVA algorithms ([8]).

4. Conventional Multiuser System

The block diagram of the conventional detector is shown in fig. 3 [9].

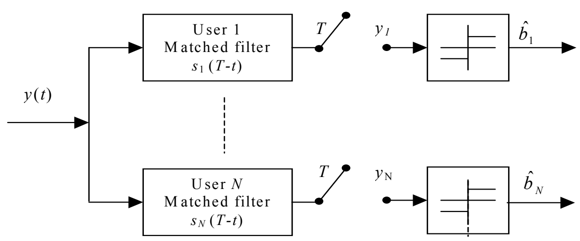

Fig. 3. The conventional multiuser detector

In multiple access communication systems the conventional detectors are composed of bank of parallel matched filters. One filter is matched for one user's signal. Their roles are to separate the signal of each user from the received signal and to estimate the transmitted bits. The output of each filter can be expressed as:

$$y_k = \int_0^T y(t) s_k(t) dt = A_k b_k + \sum_{j \neq k} A_j b_j \rho_{kj} + n_k,\ k = \overline{1, N} \tag{3}$$

$$n_k = \sigma \int_o^T n(t) s_k(t) dt,\quad k = \overline{1, N} \tag{4}$$

is a Gaussian random variable with zero mean and variance σ^2. In matrix representation eq. (3) becomes:

$$\mathrm{Y} = \mathrm{Rab} + \mathrm{N} \tag{5}$$

where $\mathrm{Y} = [y_1, y_2, \ldots y_N]^T$: column vector that includes the outputs of the matched filters; $\mathrm{A} = diag\{A_1, A_2, \ldots A_N\}$: diagonal matrix of the amplitudes of the received bits; $\mathrm{b} = [b_1, b_2, \ldots b_N]^T$: column vector containing bits received from users; $\mathrm{N} = [n_1, n_2, \ldots n_N]^T$ is the sampled noise vector; **R** is the cross-correlation matrix in eq. (2).

The estimated bit, after the threshold comparison, is

$$\hat{b}_k = \mathrm{sgn}(y_k) = \mathrm{sgn}\left(A_k b_k + \sum_{j \neq k} A_j b_j \rho_{kj} + n_k \right) \tag{6}$$

The error probability is, thus, influenced by the noise samples, correlated with the spreading sequences and by the interference from the other users.

5. MMSE Multiuser Detector

The linear mean square multiuser (MMSE) detector uses a linear combination of the matched filter outputs that minimizes the mean square error between these outputs and the correspondent transmitted data. For each user *k*, the detector has to determine a vector of *N* weighting coefficients m_k that maximize

$$E\left[(b_k - m_k^T y)^2\right] \tag{7}$$

The general scheme of such detector is represented in fig. 4:

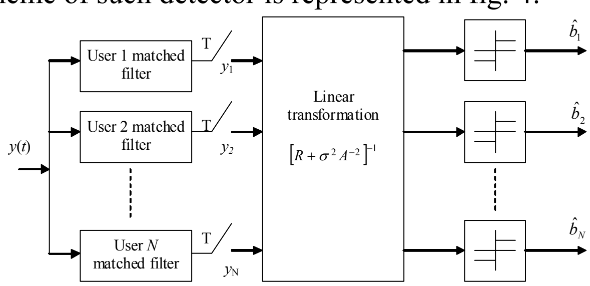

Fig. 4. Linear MMSE detector

Taking into account all users and using the matrix representation (5), this leads to

$$\min_{M \in R^{N \times N}} E\left[\|b - MY\|^2 \right] \tag{8}$$

where M is an $N \times N$ matrix whose *k* column is equal to m_k. Algebraic calculations lead to the solution

$$M^{*} = \left[R + \sigma^{2} A^{-2} \right]^{-1} \tag{9}$$

where it has been assumed that A is a non-singular matrix (i.e. only those users that are active are taken into consideration) [10].

Extracting the column corresponding to each user, the estimated bit associated to user k is:

$$\hat{b}_k = \operatorname{sgn}\left(\frac{1}{A_k} \left(\left[R + \sigma^2 A^{-2} \right]^{-1} y \right)_k \right) = \operatorname{sgn}\left(\left(\left[R + \sigma^2 A^{-2} \right] y \right)_k \right) \tag{10}$$

6. Optimal Multiuser System

The optimal multiuser detector maximizes the joint *a posteriori* probability by evaluate a log-likelihood function over the set of all possible information sequences. Tree-type maximum-likelihood sequence detectors have been studied for multiuser system. In the scheme proposed in [11] a local metric plays an important role and only a subset of paths is always dealt with. Hence there is no built in mechanism to reach the global optimum. Simulated evolution (e.g., evolutionary programming (EPs) and GAs) can be used as an effective global optimization procedure [12]. The disadvantages of GAs are slow convergence to a good near-optimum and high computational complexity [13]. For example when the system is used by two users the optimum detection states that the estimated bits are:

$$\begin{cases} \hat{b}_1 = \operatorname{sgn}\left(A_1 y_1 + \frac{1}{2}\left| A_2 y_2 - A_1 A_2 \rho \right| - \frac{1}{2}\left| A_2 y_2 + A_1 A_2 \rho \right| \right) \\ \hat{b}_2 = \operatorname{sgn}\left(A_2 y_2 + \frac{1}{2}\left| A_1 y_1 - A_1 A_2 \rho \right| - \frac{1}{2}\left| A_1 y_1 + A_1 A_2 \rho \right| \right) \end{cases} \tag{11}$$

7. Simulation Results

The number of users of the system is 2. The number of bits forming the signal of one user is 500. Depending on the power of the noise on the channel and the power of the signals from the users, different BER values can be achieved. Extensive Monte Carlo simulations have been performed and the results obtained lead to several interesting conclusions. The simulation parameters for turbo technique are: frame size: 500 bits; number of iterations: 10; no puncturing; code rate: ½; both RSCs use generator polynomial of degree 2 ([1 1 1; 1 0 1]) and random interleaver for both coding and decoding algorithms.

For multiuser detection schemes the parameters used include:

- vector of (non) equal amplitudes of received signals :

$$A = [3\ 3] \ \ / \ \ A = [1.5\ \ 4] \tag{12}$$

- orthogonal spreading sequences:

$$S_1 = [111-1111-1]/\sqrt{8}$$
$$S_2 = [11-1111-11]/\sqrt{8} \qquad (13)$$

- non-orthogonal spreading sequences:

$$S_1 = [1\ \ -1\ \ -1\ \ 1\ \ \ 1\ \ -1\ \ \ 1-1]/\sqrt{8}$$
$$S_2 = [1\ \ -1\ \ \ \ 1-1-1\ \ \ 1-1\ \ \ 1]/\sqrt{8} \qquad (14)$$

- turbo decoding algorithm used is MAP since we have already proved in previous papers that this algorithm leads to similar performances to SOVA algorithm and its implementation of is simple.

I. *Conventional Multiuser Detector*

A. Signals with equal powers and orthogonal spreading sequences

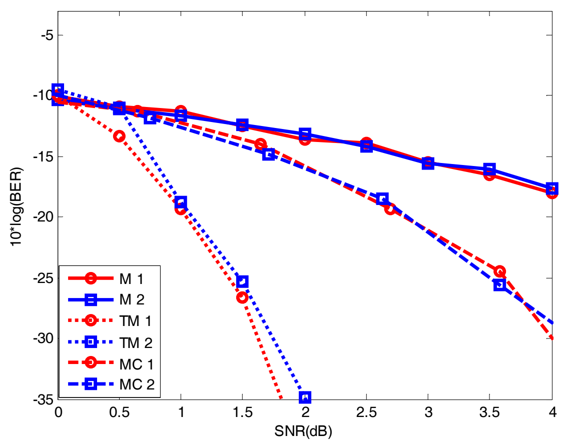

Fig. 5. BER vs SNR for conventional detectors with equal powers, orthogonal sequences

From fig.5 several conclusions can be highlighted:

- For variation of SNR between 0 and 4 dB there are no significant changes in the behavior of the system from BER point of view. The reason is that the simulations have been performed in almost ideal. Convolutional techniques lead to better results than the simple system does and performances increase as SNR increases;
- For SNR= 0 dB all techniques illustrate the same performances. BER reaches –9.93dB which represents a good performance of the system.
- Due to the very good decoding algorithm of turbocodes technique BER curve for conventional multiuser detector decreases very fast for the studied SNR

interval unlike the simple detector whose performances decrease slowly. The performances of 2 users conventional detector are the same for both users meaning that turbo technique compensates very well the influence of the second user;

- Between simple, convolutional and turbo detector the best performances are achieved in the case of turbo technique usage. Convolutional technique can improve this system's performances only for large values of SNR.

B. Signals with equal powers and non-orthogonal spreading sequences

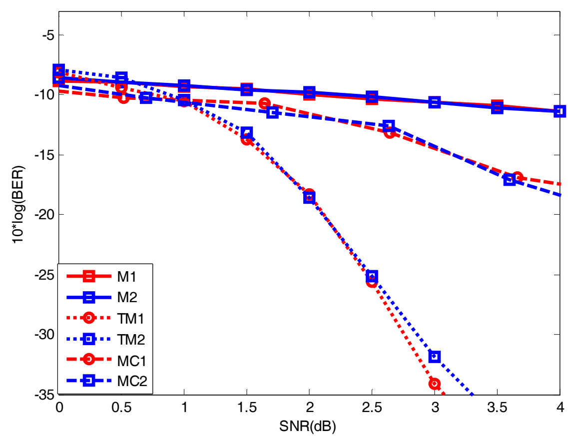

Fig. 6. BER vs SNR for conventional detectors with equal powers of signals, non-orthogonal sequences

In fig. 6 there are shown simulation results for conventional system in combination with convolutional/turbo technique in case of equal amplitudes of the signals and non-orthogonal spreading sequences.

- In case of simple conventional detector for SNR= 0 dB we obtain a BER≈ -8dB which indicates that this type of detector without an appropriate coding/decoding technique leads to lower performances of the system than the ones shown in the previous case;
- Unlike simple detector the ones that use convolutional technique and turbo technique illustrates significant improvement of performances of the system; this way for SNR = 0 dB → BER = -9 dB, value encountered also in the results of the system using orthogonal sequences. Although the performances of system using simple detector and convolutional detector are resembling the coding gain in the second case is approx. 6dB;
- For conventional detector values of BER decrease slower than in the case of orthogonal sequences usage; for turbo conventional detector BER decreases for

SNR around 1 dB and then the curve of BER show the same tendency like the case of cross-correlation equal to 0 (orthogonal sequences).

- As cross-correlation strengthens values for BER increase especially for conventional detector. It can be stated that this type of detector can be used only when the spreading sequences are orthogonal or cross-correlation values are small. Instead turbo conventional detector can be used for large correlation values. BER reaches -35 dB for SNR = 3 dB.

C. Signals with non- equal powers and orthogonal spreading sequences

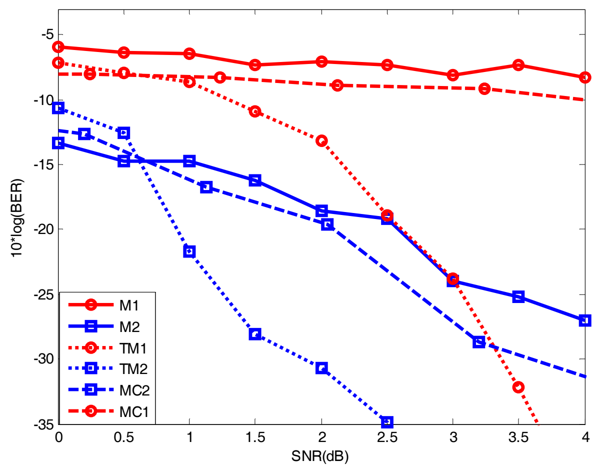

Fig. 7. BER vs SNR for conventional detectors with non-equal powers of signals, orthogonal sequences

From fig.7 it can be noticed:

- For conventional detector the user transmitting with the lowest power 3.5 dB, BER has a behaviour almost linear increasing from SNR=0dB to SNR=4dB with only 2.41dB.
- Using convolutional technique the performances are not improved significantly. It can be said that this technique is not appropiate for this transmission since it doesn't compensate the disadvantage of different powers for the users signals;
- In case of turbo technique the results overcome the ones from the other detectors. For user 1 with the signal's power of 3.5 dB (1.5V) the increase of BER is approx. 6dB for SNR = 2 dB and for SNR = 4 dB BER increases faster with 34.77dB.
- For the user with higher signal power values of BER are not different than the ones achieved in case of equal signals power. This can be translated in a good

compensation of effect given by the user with higher power but still the effect of user with lowest signal power (under 9 dB) cannot be eliminated.

II. *Optimum Multiuser Detector*

A. Signals with equal powers and orthogonal spreading sequences

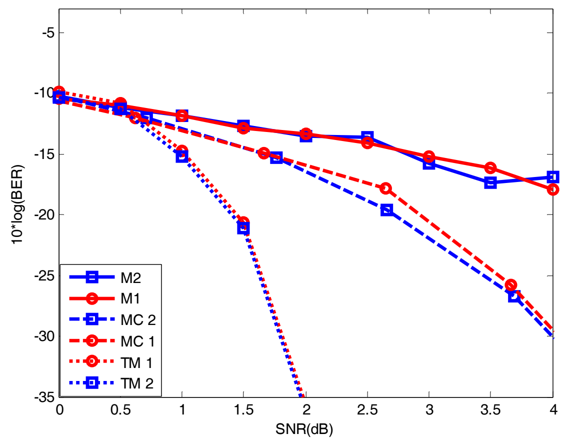

Fig. 8. BER vs SNR for optimum detectors with non-equal powers of signals, orthogonal sequences

The following observation can be made from fig. 8:

- For simple, convolutional and turbo techniques BER starts from -10 dB at SNR=0 dB but the variation of BER curve for convolutional and turbo systems is stronger;
- As expected BER decreases slow for simple detector. For SNR between (0-4) dB, BER variation is approx. 5 dB;
- According to the speciality literature the performances of optimum detector are identical to the performances of conventional detector in almost perfect conditions.

B. Signals with equal powers and non-orthogonal spreading sequences

The performances of systems when equal amplitude signals and non-orthgonal sequences are used **(**fig. 9**)** can be comment as:

- In spite of the fact that both users are strongly correlated BER remains almost the same as in the case of zero cross-correlation coefficients both fot simple and turbo system;

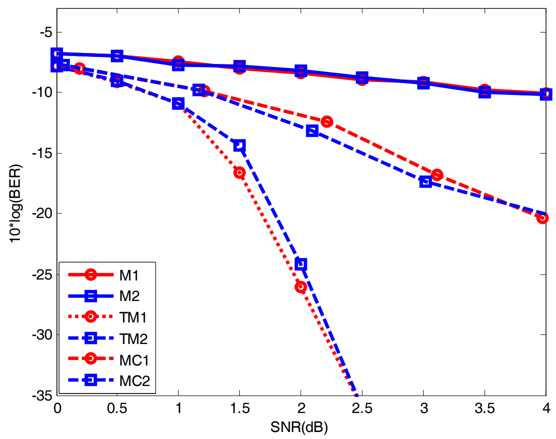

Fig. 9. BER vs SNR for optimum detectors with equal powers of signals, non-orthogonal sequences

- For simple optimum detector BER decreases from -6.81 dB for SNR = 0 dB to -10 dB for SNR = 4 dB and for turbo optimum detector BER = -7.78 dB for SNR = 0 dB reaching -35 dB for SNR = 4 dB;
- Usage of convolutional technique brings at least 10 dB gain for BER in case of RSZ= 4dB meaning that a coding/decoding technique increases significantly the performances of the system;
- Convolutional technique manage to compnesate the efect of different correlation coefficients so the performances of this system can be asimilated as a system with one user in identical conditions;
- Turbo technique corrects errors occuring due to cross-correlation between users as long as this value remains between certain limits.

C. Signals with non-equal powers and orthogonal spreading sequences

The vector of amplitudes is [1 2] corresponding to 0 dB for user 1 and 6 dB for user 2. From fig. 10 it can be noticed:

- For user 1 turbo optimum detector leads to the same performances as simple optimum detector starting from BER = -5.46 dB for SNR = 0 dB and reaching BER = -12.29 for SNR = 4 dB with a slow decrease;
- For user 2 transmitting signal with high power in turbo optimum detector the curbe of BER decreases from -8.42 dB for SNR = 0 dB to -52.28 dB for SNR = 4 dB. The more the power of the signal increases the more turbo optimum detector approaches the performances of simple turbo system managing to eliminate the effect of the user transmitting signal with higher power.

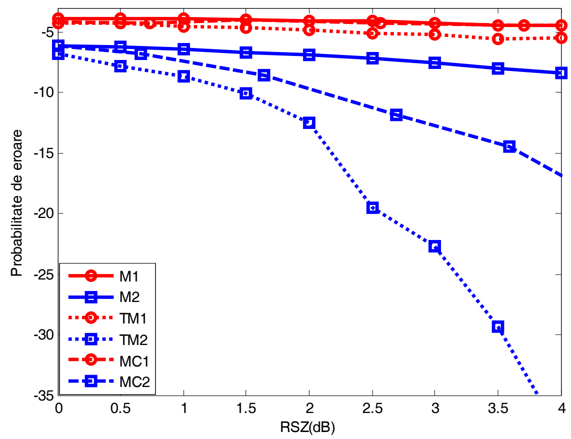

Fig. 10. BER vs SNR for optimum detectors with non-equal powers of signals, orthogonal sequences

III. *MMSE Multiuser Detector*

Simulations have been performed for MMSE detector structures with 2 users, having orthogonal/non-orthogonal sequences, equal and different amplitudes. The data can be also convolutional/turbo-encoded and decoded. The results will be compared in terms of BER versus SNR.

A. Signals with equal powers and orthogonal spreading sequences

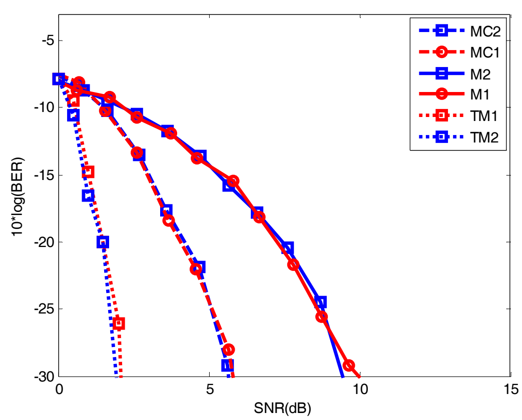

Fig. 11. BER vs. SNR for MMSE detectors with equal powers of signals, orthogonal sequences

- Considering the ideal conditions for this communication system (equal amplitudes and orthogonal sequences) the performances for 2 users are similar to the performances of 1 user system. Simple MMSE detector achieved poor performances for small BER values reaching BER= -30 dB for SNR= 10 dB. Compared to this value the best performances are obtained in case of turbo technique usage in addition to MMSE system. For this system the same value for BER is achieved at SNR value approx. 2 dB. Even though a cross-correlation coefficient equal to 0 suggest signals not correlated the performances achieved are very good this is not applying to turbo MMSE detector.
- The performances of the system are improved when convolutional technique is used but still they are not close to the turbo detection system. BER= -30 dB is obtained for SNR=-5 dB showing improvement in performance for simple detection system.
- Therefore it is obvious that resistance to noise of linear MMSE detector is good when users transmit signals with equal power. Still the performances achieved are similar to the performances of conventional and optimum detector in the same environment.

B. *Signals with equal powers and non-orthogonal spreading sequences*

From fig. 12 it can be said:

- From cross-correlation matrix it can be observed the two signals are correlated and this correlation coefficient is 0.5.

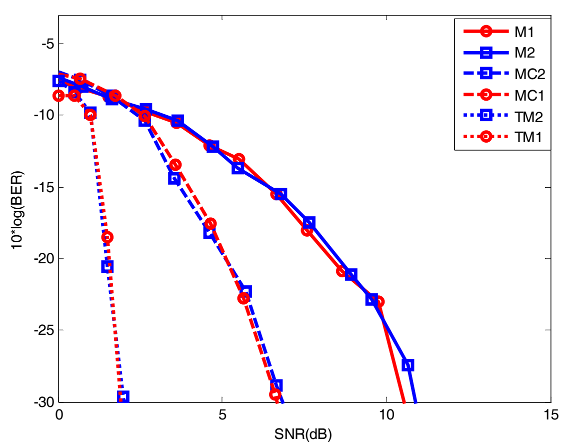

Fig. 12. BER vs. SNR for MMSE detectors with equal powers of signals, non-orthogonal sequences

- These performances obtained are similar to the performances obtained in the first case meaning this system can compensate the cross-correlation effects on the performances and the effect of nonequal amplitudes (shown in the next case). Yet the performances are only less than 1 dB worse than the once achievedin ideal case. Because the signals are correlated BER curve has the same shape when signals have equal powers.

- It is very important to mention that turbo technique usage annuls strong correlation between signals and we have a value of -35 dB for SNR=2 dB. In case of linear MMSE when correlation was increasing BER increased also but this disadvantage has been eliminated using turbo coding/decoding.

C. Signals with nonequal powers and orthogonal spreading sequences

- Even though the power of signals is different and the spreading sequences are orthogonal (there is no correlation between users signals) the dependence of BER vs. SNR is approx. the same as in previous cases. For SNR values between (0-5) dB BER curve tendancy is identical to the one in fig.9. An important decrease is noticeable for high SNR values between (5-10) dB. For this system at SNR=6dB the coding gain is 2dB.

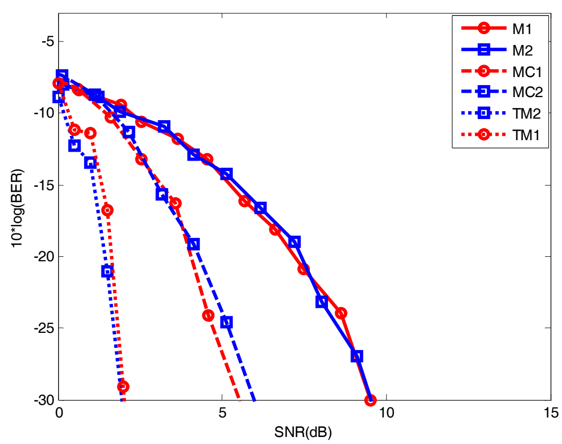

Fig. 13. BER vs. SNR for MMSE detectors with nonequal powers of signals, orthogonal sequences

- Although users are transmitting signals with different powers by using a MMSE detector the effect of the user transmitting signal with lower power is attenuated but only when orthogonal spreading sequences are used.
- Compared to the other cases studies the results are obvously superior to convolutional systems or simple systems performances. For turbo MMSE system the minimum BER reaches -55.68 dB (for SNR= 4 dB). Even in critical conditions when signal's level equals noise level the turbo techniques corrects numerous errors and therefore a BER equal to approx. 10 dB can be achieved.

8. Conclusions

The purpose of this research is to determine the most performant multiuser detector that can be used to recover signals at base station levels. As shown in the paper without an appropriate coding/decoding technique the performances of such system are poor and they can be improved by carefull selection of spreading sequences, power of the signals, etc.

Our first conclusion is that turbo technique is a very powerful and good solution for overcoming the disadvantages that might appear in a communication line. The main advantage of this technique compared to convolutional technique is the usage of an interleaver block within its structure and the iterative decoding algorithms. There are two very performant turbo decoding algorithm: MAP şi SOVA. The choice of such algorithm is left at users desire.

For all types of detectors we concluded that for strongly correlated users the performances of the system are identical and similar to the case of one user system. As cross-correlation coefficient increases the performances are obviously degraded. For SNR value between 0 and 4dB and for stronlgy correlated users BER decreases with almost 6 dB. Instead for not correlated users the curve of BER for the same SNR values decreases with is only 2dB.

The analysis of signals power influence led to the conclusion that the higher the power of transmitted signal, the better the performances of the system are. For a selected SNR range BER curve for the user transmitting signal with lower power decreases with only 2 dB unlike for user with higher power signal 10 dB.

The disadvantage of users transmitting signals with different powers is compensated by turbo coding/decoding technique for all type of users. The effect is still more obvious in the case of MMSE and optimum detector since they manage to eliminate this effect very fast even for SNR values high.

Combining the two effects of cross-correlation degree and power of users signals we achieved the best results for all detectors in case of orthogonal spreading sequences usage and equal power of signals. The more we modify the power of the signals or the spreading sequences used we estrange from the ideal case.

When non-orthogonal spreading sequences are used additional coding/decoding techniques become very useful in non-orthogonality effects. This affects significantly the performances of the systems at the base station level.

Previous simulations have demonstrated that transmission of signals with different power (therefore different amplitudes) play an important role but not a determinant one when it comes to performances of the system when several coding/decoding technique are used. The importance of these unequal powers appears when the system is not using any technique to suppress the negative effects.

Our future work aims:

- to increase the number of users of the system: the final goal is to implement these detectors at base station level. Regulary this base station is accessed by a large number of users so the necessity is obvious.
- to increase the number of iteration so the number of errors that might appear to decrease. This effect is transposed in better performances of the systems.
- to design and implement other type of detectors such as decision/driven multiuser detectors and decorrelating detectors. These detectors may be used in asynchronous CDMA systems.

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