NEW METHOD FOR MULTICRITERIA ANALYSIS

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*Starting with a classical multi criteria analysis problem: the alternatives $a_1, a_2, \ldots, a_m$ to be appraised by the criteria $c_1, c_2, \ldots, c_n$, we make one finite partially ordered set with defined new alternatives dependent on the given ones (better and weaker) with weight of paths. So, by given procedure, we have the best alternative (**1** in poset) and the worst alternative (**0** in poset). For all $i \in \{1, 2, \ldots, m\}$, we define the distance of the alternative a_i from **1** and the distance of **0** from the alternative a_i. Using those distances we make the order of alternatives.*

Keywords: Partially ordered set, Alternative, Criterion.

1. Introduction

Multi criteria analysis is a decision-making tool developed for complex problems. In a situation where multiple criteria are involved confusion can arise if a logical well-structured decision-making process is not followed. Another difficulty in decision making is that reaching a general consensus in a multidisciplinary team can be very difficult to achieve.

Application of a multi criteria analysis could support policy makers in choosing the control strategy that meets best all these conflicting interests. A multi criteria analysis can be effective in increasing understanding, acceptability and robustness of decision problems. It generally improves the quality and transparency of the decision making process.

The multi criteria analysis technique deals with complex problems that are characterized by any mixture of quantitative and qualitative objectives, by breaking the problem into more manageable pieces to allow data and judgments.

The applied multi criteria analysis involved the following steps: establish the decision context, identify the alternatives to be appraised, identify objectives and criteria, score, weight, calculate overall value, examine the results and sensitivity analysis.

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There are a number of developed methods and techniques of solving such problems.

Methods for determining non-inferior solutions define their set, and decision-maker is able to adopt a final resolution. By the method of weight coefficients (N.O. Da Cunha and F. Polak [11], H.W. Kuhn and A.W. Tucker [23]), as one of the oldest method of multi-criteria optimization, solutions are obtained by solving the scalar task (L.A. Zadeh [23] and M. Zeleny [24]), an algorithm of multicriteria simplex method for the determination of non-inferior solutions is shown. Non-inferior set of solutions J. Cohon and D. Marks [10] and Y.Y. Hames, W.A. Hall and H.T. Freedman [19] is obtained by the method of limits in the space of criterion functions.

By interactive methods, preference is expressed in different stages. The authors R. Benayoun, J. De Montgolfier, J. Tergny and O. Lazitchev [3] gradually express their preference by the STEM (Step Method). D.E. Monarcji, C.C. Kisiel and L. Duckstein [27] introduce SEMOPS (Sequential Multiple Objective Problem Solving) approach.

Stochastic method introduces the concept of decision making with uncertainty in terms of preference (Y.Y. Haimes [18], L. Duckstein, D. Monarchi and C.C. Kisicl [12]). A. Goicoechea, D.R. Hansen and L. Ducktein [15] analyzed the probability of achieving the criterion function value using PROTRADE (Probabilistic Trade-off Development) method.

The method presented in this paper belongs to the group of methods with a preference marked in advance. Other authors, e.g. R.L. Keeny and A. Sicherman [22] solved the problem of multi-criteria decision-making by "Utility theory." Papers by A. Charnes and W.W. Cooper [9]; F.W. Gembicki and Y.Y. Haimes [14] and J.P. Ignizio [21] describe the application of targeted programming, actually finding of a possible solution that is closest to a given goal. ELECTRA method resulting graph is obtained by examining the degree of agreement between weight preference, as well as disagreement of certain actions weight (B. Roy [30]). Later versions of methods, ELECTRE II (B. Roy et B. Bertier (1972)), ELECTRE III and IV (B. Roy and J.C. Hugonnard [33]), made significant improvements in the level of getting complete preferences, and other.

PROMETHEE method (Preference Ranking Organization Enrichment Methods for Evaluation) was developed by J.P. Brans and P.A. Vincke [6]. In this method all pairs of existing actions are considered separately for each criterion, where the authors introduce functions of preferences. They are mathematical functions by which decision-makers can express the intensity and the limits of their preferences on specific criteria. The authors have developed four variations of these methods: PROMETHEE I gives partial order of alternatives, PROMETHEE II provides a complete order, PROMETHEE III gives an interval order, and PROMETHEE IV is used for continuous set of alternatives.

With respect to a large number of approaches and mathematical models in multi-criteria optimization (C.M. Brugha [7]), we would note some of them: Extension of PROMETHEE method through fuzzy mathematical programming (A.S. Fernández-Castro and M. Jiménez [13]), Representation of multi-criteria analysis by means of artificial intelligence techniques (G. Balestra and A. Tsoukiàs [1]), A Bayesian approach for multiple criteria decision making (R. Rajagopal and E. Del Castillo [29]).

2. A new method

Consider the *set of alternatives* $A=\{a_1,a_2,\dots,a_m\}$ that are evaluated by the *set of criteria* $C=\{c_1,c_2,\dots,c_n\}$.

We believe that decision makers have clearly established the criteria that are relevant for evaluating alternatives. By the procedure that will be presented in this paper we will assume that all criteria don't have to be of the same importance and that the decision makers are aware of the degree of importance of each criterion. Thus it is possible to evaluate the alternatives by multiple criteria, where the final score is to be formed on the basis of criteria that are essential for the ranking of alternatives as well as of the criteria of less importance.

So, we will assume that each criterion c_k, $k\in\{1,2,\dots,n\}$, is associated with a number $z_k\in(0,1]$ that represents the degree of importance of that criterion when evaluating a given set of alternatives.

Also, for each criterion we will consider the function that measures the importance of the difference between two given alternatives. For example, if someone wants to buy a car, the difference in price of 500 € between two cars at a cost of 11000 € and 11500 € do not play a major role, while that difference is quite important when deciding between two cars at a cost of 2000 € and 2500 €.

These functions, in fact, will be determined by certain common agreement of one who makes decisions and one who makes calculation, and they are secret to the outside world.

Therefore, we will consider the specific functions $\mathscr{Z}_k : A\times A\to[0,+\infty)$, for each criterion c_k, $k\in\{1,2,\dots,n\}$, which joins a nonnegative real number $\mathscr{Z}_k(a_i,a_j)$ to each pair of alternatives $a_i,a_j\in A$.

For convenience we will assume that for any $i\in\{1,2,\dots,m\}$, the alternative a_i is represented as an n-tuple $a_i=(a_{i1},a_{i2},\dots,a_{in})$, or equivalently $a_i=(a_{ik})_{k=1}^{n}$, where each coordinate a_{ij}, $j\in\{1,2,\dots,n\}$, is nonnegative real number which represent the degree of satisfaction of the criterion c_j.

Now we will create two new sets of (hypothetical) alternatives in the following way

$$\overline{A} = \{(\max_{i\in J} a_{ik})_{k=1}^{n} \mid J \in \mathscr{P}(\{1,2,\dots m\}) \setminus \varnothing\}, \tag{1}$$

$$\underline{A} = \{(\min_{i\in J} a_{ik})_{k=1}^{n} \mid J \in \mathscr{P}(\{1,2,\dots m\}) \setminus \varnothing\}. \tag{2}$$

The set $\overline{A}$ is closed under max-operation, i.e., for every $a,b \in \overline{A}$ holds

$$\max\{a,b\} = (\max\{a_k, b_k\})_{k=1}^{n} \in \overline{A}. \tag{3}$$

The natural order relation on $\overline{A}$ is defined as follows

$$a \le_{\overline{A}} b \iff \max\{a,b\} = b, \quad \text{for every } a,b \in \overline{A}. \tag{4}$$

$(\overline{A}, \le_{\overline{A}})$ is an upward semilattice with the greatest element $\max A$ which will be denoted by $\mathbf{1}$ and which will be called the *best alternative*.

The set $\underline{A}$ is closed under min-operation, i.e., for every $a,b \in \underline{A}$ holds

$$\min\{a,b\} = (\min\{a_k, b_k\})_{k=1}^{n} \in \underline{A}. \tag{5}$$

The natural order relation on $\underline{A}$ is defined as follows.

$$a \le_{\underline{A}} b \iff \min\{a,b\} = a, \quad \text{for every } a,b \in \underline{A}. \tag{6}$$

$(\underline{A}, \le_{\underline{A}})$ is an downward semilattice with the least element $\min A$ which will be denoted by $\mathbf{0}$ and which will be called the *worst alternative*.

Then, on the set $L = \overline{A} \cup \underline{A}$, we define a partial order preserving the orders on $\underline{A}$ and $\overline{A}$, i.e., for $a,b \in L$ if $a \le_{\overline{A}} b$ then $a \le b$ and if $a \le_{\underline{A}} b$ then $a \le b$. In fact, $(L, \le)$ is a partially ordered set regarded as the sum of downward and upward semilattices.

Let us recall that an element b of partially ordered set L *covers* $a \in L$, which will be denoted by $a \prec b$, if $a < b$ and $c \in L$ such that $a \le c \le b$ implies $c = a$ or $c = b$.

Let $a,b \in L$ be such that $a \prec b$. Then for every criterion c_k, $k \in \{1,2,\dots,n\}$, we define the *preference of the alternative* b *over the alternative* a *with respect to the criterion* c_k as follows

$$\delta_k(a,b) = \mathscr{K}_k(a,b) \cdot \frac{b_k - a_k}{\mathbf{1}_k - \mathbf{0}_k}. \tag{7}$$

We will use the following notation

$$a \overset{\delta_k(a,b)}{\longmapsto} b.$$

Let $a, b \in L$ and let P be a path in L from a to b, i.e., there exis $p_1, p_2, \ldots, p_j \in L$ such that

$$P:\ a = p_1 \overset{\delta_k(p_1,p_2)}{\longmapsto} p_2 \overset{\delta_k(p_2,p_3)}{\longmapsto} \cdots \overset{\delta_k(p_{j-1},p_j)}{\longmapsto} p_j = b\,. \tag{8}$$

Then the *running preference of the alternative* b *over the alternative* a *with respect to the criterion* c_k *through the path P* is defined as

$$\delta_k^P(a,b) = \begin{cases} 0, & \text{if } \delta_k(p_i, p_{i+1}) = 0 \text{ for all } i \in \{1,\ldots,j\}, \\ \dfrac{1}{\sum_{i=1}^{j-1} \mathscr{Z}_k(p_i, p_{i+1})} \cdot \sum_{i=1}^{j-1} \delta_k(p_i, p_{i+1}), & \text{otherwise,} \end{cases} \tag{9}$$

and according to (7) we have

$$\delta_k^P(a,b) = \begin{cases} 0, & \text{if } \delta_k(p_i, p_{i+1}) = 0 \text{ for all } i \in \{1,\ldots,j\}, \\ \dfrac{1}{\sum_{i=1}^{j-1} \mathscr{Z}_k(p_i, p_{i+1})} \cdot \sum_{i=1}^{j-1} \mathscr{Z}_k(p_i, p_{i+1}) \cdot \dfrac{p_{i+1,k} - p_{ik}}{\mathbf{1}_k - \mathbf{0}_k}, & \text{otherwise.} \end{cases} \tag{10}$$

Starting with alternative a, the alternative b can be reached via several different paths in L with corresponding running preference weights, so we will define the *preference of the alternative* b *over the alternative* a *with respect to the criterion* c_k as follows:

$$\mathscr{P}_k(a,b) = \max_{P:\, a \to b} \delta_k^P(a,b)\,, \tag{11}$$

and therefore, the *preference of the alternative* b *over the alternative* a can be regarded as n-tuple

$$\mathscr{P}(a,b) = \left(\mathscr{P}_k(a,b)\right)_{k=1}^n\,. \tag{12}$$

The *distance of alternative* $a \in L$ *from the best alternative* $\mathbf{1}$ is defined as

$$\mathscr{D}_1(a) = \frac{1}{\sum_{k=1}^n z_k} \cdot \sum_{k=1}^n z_k \cdot \mathscr{P}_k(a, \mathbf{1})\,, \tag{13}$$

and the *distance of the worst alternative* $\mathbf{0}$ *from the alternative a* is defined as

$$\mathscr{D}_0(a) = \frac{1}{\sum_{k=1}^n z_k} \cdot \sum_{k=1}^n z_k \cdot \mathscr{P}_k(\mathbf{0}, a)\,. \tag{14}$$

Now, for two alternatives $a, b \in A$, we say that the alternative a *prefers* the alternative b if a is closer to the best alternative and the worst alternative is further of a, i.e.,

$$\begin{aligned} a \textit{ prefers } b \text{ if and only if } & \mathscr{D}_1(a) < \mathscr{D}_1(b) \text{ and } \mathscr{D}_0(b) > \mathscr{D}_0(b), \\ \text{or } & \mathscr{D}_1(a) = \mathscr{D}_1(b) \text{ and } \mathscr{D}_0(a) > \mathscr{D}_0(b), \\ \text{or } & \mathscr{D}_1(a) < \mathscr{D}_1(b) \text{ and } \mathscr{D}_0(a) = \mathscr{D}_0(b), \end{aligned} \tag{15}$$

and we say that the alternative a is *indifferent* over the alternative b if

$$\mathscr{D}_1(a) = \mathscr{D}_1(b) \text{ and } \mathscr{D}_0(a) = \mathscr{D}_0(b), \tag{16}$$

otherwise, we say that the alternatives a and b are *incomparable*.

This relation offers a graph to decision makers, in which some alternatives are comparable and some are not. This information can be useful in practical applications.

If decision makers want to have a total order of alternatives, then they can form the following relation, which gives full rank of alternatives, but this rank is poorer with information and less realistic because it comes to balancing distances of the best and the worst alternative.

For alternative $a \in A$, we define the *difference*

$$\mathscr{D}(a) = \mathscr{D}_0(a) - \mathscr{D}_1(a). \tag{17}$$

A total order of alternatives is defined as follows. For $a, b \in A$,

$$a \textit{ prefers } b \text{ if and only if } \mathscr{D}(a) > \mathscr{D}(b), \tag{18}$$

$$a \textit{ indifferent } b \text{ if and only if } \mathscr{D}(a) = \mathscr{D}(b). \tag{19}$$

3. Examples

Example 1. This is one hypothetical example in which we will show the previously described method graphically. Let us observe alternatives $A = \{a_1, a_2, a_3, a_4\}$ and let appraise them by a set of criteria $C = \{c_1, c_2, c_3, c_4\}$, as shown in Table 1.

Table 1

	c_1	c_2	c_3	c_4
a_1	2	2	70	80
a_2	2	3	65	80
a_3	3	2	65	75
a_4	4	2	60	80

Let for all $k \in \{1, 2, \ldots, n\}$, the functions $\mathscr{Z}_k : A \times A \mapsto \mathbb{R}^+$ be defined as follows

$$\mathscr{Z}_k(a,b) = \begin{cases} 1, & \text{if } a_k \neq b_k, \\ 0, & \text{otherwise,} \end{cases} \tag{20}$$

and let $z_1 = z_2 = z_3 = z_4 = 1$, i.e., let all criteria have the same relative importance (equal to 1). Then, the partially ordered set of "new alternatives" is given in Fig. 1.

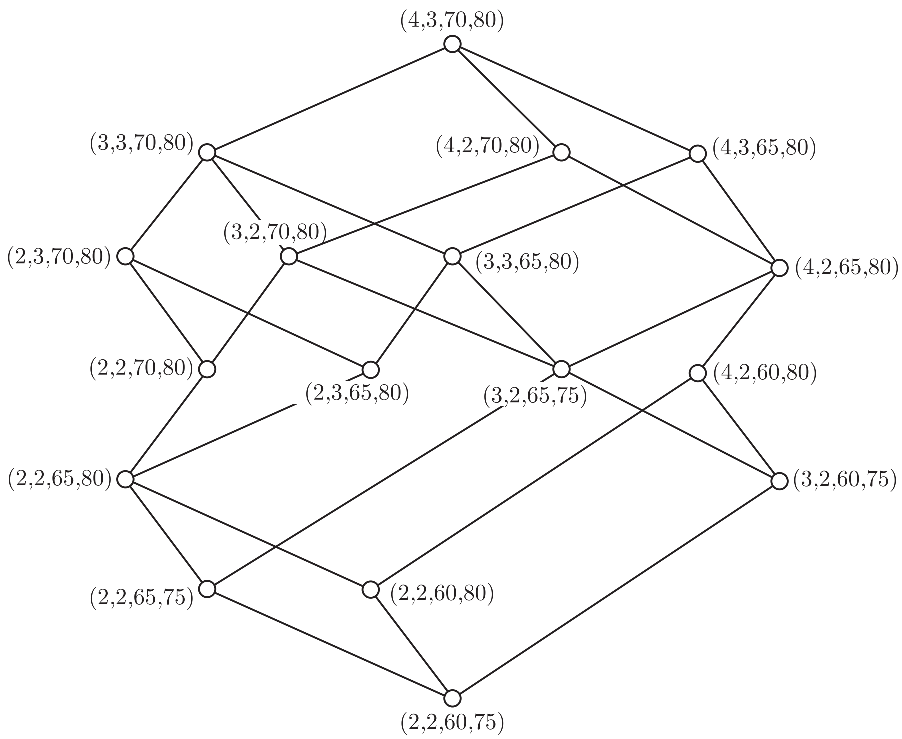

Fig. 1. The partially ordered set of "new alternatives"

The corresponding distances are presented by Table 2, which induce the graph given in Fig. 2. So, the alternative a_2 is the best choice, the alternatives a_1 and a_4 are indifferent, i.e., they are equally good choice, and a_3 is the worst choice among all of them. Notice here that the same rank of alternatives can be obtained by Promethee method or by compromise ranking method.

Table 2

	a_1	a_2	a_3	a_4
$\mathscr{D}_1$	0.5	0.375	0.75	0.5
$\mathscr{D}_0$	0.5	0.625	0.25	0.5
$\mathscr{D}$	0	0.25	–0.5	0

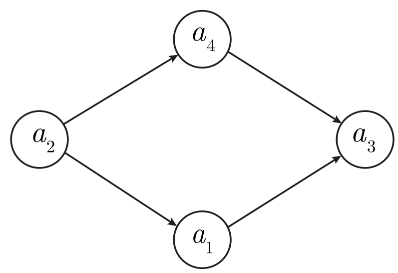

Fig. 2

Example 2. Let us observe the set of alternatives A with respect to the set of criteria C given in Table 3.

Table 3

	c_1	c_2	c_3	c_4	c_5
a_1	100	4	32	50	4.5
a_2	95	3	35	44	4.4
a_3	90	6	24	40	4.0
a_4	70	10	30	45	4.0
a_5	150	2	33	50	5.0
a_6	120	5	32	40	6.0

By using compromise ranking method (see [28]), we obtain rank of alternatives given in Fig. 3.

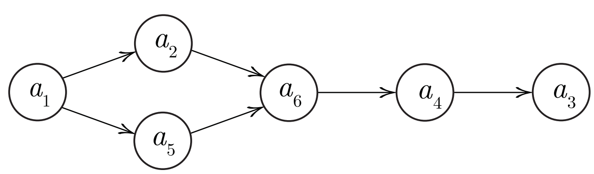

Fig. 3

It is clear that alternatives a_2 and a_5 are indifferent, i.e., they are equally good choices by this valuation method. If we look at the same set of alternatives and rank them by the method proposed in this paper, then we have the rank of alternatives given in Fig. 4.

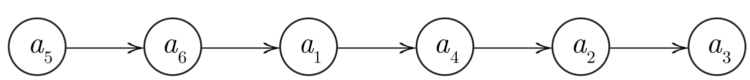

Fig. 4

As we can see in this example, the new method proposed in this paper and the compromise ranking method induce two different order relations on the same set of alternatives. This proves that this method and compromise ranking method are different, so we have an option to apply a new method when the compromise ranking method doesn't give enough information, i.e., when observed alternatives are indifferent, as it was the case with alternatives a_2 and a_5. In this particular case, by our method we have that there are no indifferent alternatives, i.e., alternative a_5 is better choice.

4. Conclusion

The proposed method for calculating the preference of one alternative over another, regarding a given criterion, offers the possibility to include in the preference both the difference between these alternatives and the values of alternatives themselves. In this way one can get fruitful and more accurate information on the observed alternatives and corresponding preference.

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