ON THE IMPLICATIONS OF THE BIOLOGICAL SYSTEMS FRACTAL MORPHO-FUNCTIONAL STRUCTURE

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Considering that the biological systems structural units dynamics are achieved on fractal curves, in the scale relativity hydrodynamic variant (with constant arbitrary fractal dimension), fractal logical elements (fractal bit, fractal cellular neural network etc.) are defined. Assuming that the external scalar potential is proportional with the fractal states density, the one-dimensional solution with finite fractal "energy" is obtained in the form of a fractal kink, whose "topology" implies, through its induced topological charge, the fractal bit. By mapping the one-dimensional solution with infinite fractal "energy", the fractal cellular neural network is obtained. In a particular case, for motions on Peano curves at Compton scale, the quantum logical elements are obtained once more. Some implications on the fractal morpho-functional structure of the lung, using this model, are shown.

Keywords: biological systems, fractals, Scale Relativity Theory, fractal bit, quantum bit, fractal cellular neural network.

1. Introduction

The biological systems dynamics imply self-organization and various chaos transition scenarios (intermittency, quasi-periodicity, sub-harmonic bifurcation etc.) [1][2] of its structural units. Thus, the collective behavior (pattern generation through "biological structural units" coherence [3][4][5]) can be mimed by self-organization, while the plasticity (functional substitution) can be mimed by various chaos transition scenarios [5].

Considering the facts above, we admit that for large temporal scales with respect to the inverse of the highest Lyapunov exponent, the deterministic

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trajectories of biological structural units (for example the lung alveoli [3][4]) are substituted by a set of potential trajectories and the definite positions concept is substituted by that of probability densities. As a result, the differentiability from standard biophysics can be replaced by fractality from the Scale Relativity Theory [6][7] or by fractality from the Scale Relativity Theory with an arbitrary constant dimension [8][9]. In both of these theories we assume that the movements of biological structural units (for example the lung alveoli [3][4]) take place on fractal curves, so that all physical phenomena involved in the biological systems dynamics depend not only on the space and time coordinates but also on scales resolution [10]. As a consequence, the variables that describe the biological systems dynamics must be considered as fractal functions. Moreover, the biological structural units may be reduced to and identified with their own trajectories, so that the biological systems will behave as a special interactionless "fluid" (biological fractal fluid).

In the present paper, assuming that the biological systems structural unit's dynamics take place on fractal curves, in the hydrodynamic formulation of the Scale Relativity Theory with an arbitrary constant fractal dimension, the fractal bit, in particular the quantum bit, and the fractal cellular network are defined. In our opinion, according to the Complex Systems General Theory [1][2], every organ, for example the lung, structures its own fundamental logical elements, which may explain specific functions mimicking by stem cells injected into organs.

2. Fractal Hydrodynamics Model

Let us reconsider the fractal hydrodynamics equations in the form [8][9]:

$$\partial_t \mathbf{V}_\mathrm{D} + (\mathbf{V}_\mathrm{D} \cdot \nabla)\mathbf{V}_\mathrm{D} = -\nabla(Q+U)\,, \tag{1}$$

$$\partial_t \rho + \nabla \cdot (\rho \mathbf{V}_\mathrm{D}) = 0\,, \tag{2}$$

where $\mathbf{V}_\mathrm{D}$ is the differentiable and scale resolution independent velocity field, ρ is the states density field, Q is the specific fractal potential

$$Q = -2D^2 (dt)^{\left(4/D_F\right)-2} \frac{\Delta\sqrt{\rho}}{\sqrt{\rho}} = -\frac{\mathbf{V}_\mathrm{F}^{\,2}}{2} - D(dt)^{\left(2/D_F\right)-1} \nabla \cdot \mathbf{V}_\mathrm{F}\,, \tag{3}$$

$\mathbf{V}_\mathrm{F}$ is the non-differentiable and scale resolution dependent velocity field

$$\mathbf{V}_\mathrm{F} = D(dt)^{\left(2/D_F\right)^{-1}} \nabla(\ln\rho)\,. \tag{4}$$

U is the external scalar potential, D is the fractal – non-fractal transition coefficient, D_F is the fractal dimension, dt, by means of substitution principle is the scale resolution [5][6][7], ∇ is the gradient operator and Δ is the Laplace operator. For D_F any definition can be used [10]. Once such a definition is accepted, it has to be constant over the entire analysis of the biological system dynamics. In a particular case, for motions on Peano curves, $D_F = 2$, at Compton scale $D = \hbar/2m_0$, where $\hbar$ is the reduced Planck constant and m_0 the rest mass of the biological system structural unit, the fractal hydrodynamic equations (1)-(3) become the quantum hydrodynamic equations.

The following conclusions are obvious:

i) Any structural unit of the biological system is in a permanent interaction with the fractal medium through the specific fractal potential (3). For motions on Peano curves at Compton scale the fractal medium corresponds to the sub-quantum level [11];

ii) The fractal medium is identified with a non-relativistic fractal fluid (the fluid structural unit may be reduced to and identified with its own trajectory, i.e. its geodesics, so that the fluid will behave as a special interactionless "fluid" by means of geodesics in a fractal space) described by the specific momentum and states density conservation laws (see equation (1) and (2));

iii) The fractal velocity $\mathbf{V}_{\mathrm{F}}$ does not represent actual motion, but contributes to the transfer of specific momentum and energy concentration. This may be seen clearly from the absence of $\mathbf{V}_{\mathrm{F}}$ from the states density conservation law (2) and from its role in the variational principle [6][7]. As an immediate consequence of the facts mentioned above, regarding biological systems dynamics, we can note that, although at a macroscopic scale, in some cases, cancer cannot be observed, although cancer cells are replicating continuously at a fractal scale – the dormant stage of cancer [3][4];

iv) Any interpretation of the specific fractal potential should take cognizance of the "self" nature of the specific momentum transfer. While the energy is stored in the form of mass motion and potential energy, some is available elsewhere and only the total is conserved. It is the conservation of energy and the specific momentum that ensures reversibility and the existence of eigenstates, but also denies a Brownian motion form of interaction with an external living medium. This implies for biological systems dynamics, the possibility of reversibility, for example cancer relapse after a "successful" treatment;

v) Two types of fractal stationary states are to be distinguished:

a) Dynamic states. For $\partial_t = 0$ and $\mathbf{V}_{\mathrm{D}} \neq 0$ equations (1) and (2) give

$$\frac{1}{2}\mathbf{V}_{\mathrm{D}}^{2}+U+Q=E\,, \tag{5}$$

$$\rho\mathbf{V}_{\mathrm{D}}=\nabla\times\mathbf{f}\,. \tag{6}$$

Consequently, the sum of the specific kinetic energy $\mathbf{V}_{\mathrm{D}}^{2}/2$, external potential, U, and fractal potential, Q, is invariant, i.e. equal to the integration constant $E\neq E(\mathbf{r})$ (see equation (5)). $E\equiv\langle E\rangle$ represents the total energy of the fractal dynamic system. The states density current, $\rho\mathbf{V}_{\mathrm{D}}$, has no sources (see equation (6)), i.e. its streamlines are closed.

b) Static states. For $\partial_t=0$ and $\mathbf{V}_{\mathrm{D}}=0$, equations (1) and (2) give

$$U+Q=E\,. \tag{7}$$

The sum of the external potential, U, and fractal potential, Q, is invariant, i.e. equal to the integration constant $E\neq E(\mathbf{r})$ (see equation (7)). $E\equiv\langle E\rangle$ represents the total energy of the fractal static system. The states density conservation law (2) is identically satisfied.

3. Spontaneous symmetry breaking at fractal scale and its implications

Let us consider the static states

$$\partial_t=0\,,\ \mathbf{V}_{\mathrm{F}}=D(dt)^{\left(2/D_F\right)-1}\nabla S=0\,, \tag{8}$$

i.e. the phase coherence, $S=const.$ of the fractal fluid structural units. Then, equation (7) with the substitutions

$$U=E\rho, E=const.>0, \rho^{1/2}=g, \mathcal{D}=D(dt)^{\left(2/D_F\right)-1} \tag{9}$$

becomes

$$\frac{2m_0\mathcal{D}^2}{E}\Delta g=g^3-g\,. \tag{10}$$

In the one-dimensional case and using the notation $\xi=x(E/2m_0\mathcal{D}^2)^{1/2}$, Equation (10) takes the form:

$$\partial_{\xi\xi}g=g^3-g\,. \tag{11}$$

The equation (11) can also be obtained through the fractal variational principle $\delta\int L d\tau=0$ with $d\tau$ the fractal elementary volume applied to the fractal Lagrangean density (we extend the method from [12]):

$$L = \frac{1}{2}(\partial_{\xi} g)^2 - \Theta(g), \tag{12}$$

with the "potential":

$$\Theta(g) = \left(\frac{g^4}{4}\right) - \left(\frac{g^2}{2}\right). \tag{13}$$

Equation (11) has the solutions $g_F = 0, g_F = \pm 1$. By calculating the second derivative with respect to g of the "potential" entering (13) and substituting the above critical values into the result of this differentiation we find $\Theta_{\xi\xi}(0) = -1, \Theta_{\xi\xi}(\pm 1) = 2 > 0$. Therefore the solution $g_F = \pm 1$ is associated with the minimum "energy". Hence, the model under consideration has a double degenerated fractal vacuum state.

From (12) result both the "energy",

$$\varepsilon(g) = \int_{-\infty}^{\infty} d\xi \left[\frac{1}{2}\left(\partial_{\xi} g\right)^2 + \Theta(g)\right], \tag{14}$$

and the "energy" relative to the fractal vacuum:

$$\varepsilon(g) - \varepsilon(g_F) = \int_{-\infty}^{\infty} d\xi \left[\frac{1}{2}\left(\partial_{\xi} g\right)^2 + \frac{1}{4}(g^2 - 1)^2\right]. \tag{15}$$

Since all terms in (15) are positive and in view of the infinite limits of integration, the finiteness of the "energy" implies that at $\xi \to \pm\infty$

$$\partial_{\xi} g = 0, \frac{1}{4}(g^2 - 1)^2 = 0. \tag{16}$$

From this, it follows that at $\xi \to \pm\infty$ the function $g(\xi)$ tends to its fractal vacuum value $g_F \to \pm 1$. In order to find the explicit form of the solution of (11), we multiply it by $\partial_{\xi} g$ and subsequently over ξ, This yields:

$$\frac{1}{2}(\partial_{\xi} g)^2 = -\frac{g^2}{2} + \frac{g^4}{4} + \frac{1}{2} g_0, \tag{17}$$

where g_0 is a fractal integrate constant. From this, we have:

$$\xi - \xi^0 = \int_0^g \frac{dg}{\sqrt{\frac{g^4}{2} - g^2 + g_0}}, \tag{18}$$

where ξ^0 is the other fractal integrate constant. To this general solution corresponds for an arbitrary g_0 an infinite value of the "energy" $\varepsilon(g)$. To obtain the solution with finite "energy", we make use of the boundary conditions $g_F \to \pm 1$. From (17) it results that $g_0 = 1/2$. Replacing this value of g_0 into (18), the solution $f_k(\xi)$ of the field equation (17) with a finite "energy" is:

$$g_k(\xi) = g\left(\xi - \xi^0\right) = \tanh\left[\frac{1}{\sqrt{2}}\left(\xi - \xi^0\right)\right]. \tag{19}$$

We denote it the fractal kink solution (details on the standard kink can be found in [13]).

Combining (15) with $g_F = 1$ and g_k, we obtain the "energy" of the kink relative to the fractal vacuum:

$$\varepsilon(g_k) - (g_F) = \frac{2\sqrt{3}}{3}. \tag{20}$$

Thus, the fractal kink solution is obtained by a fractal spontaneous symmetry breaking (the fractal vacuum states are not invariant with respect to the fractal transformations group which makes invariant equation (11), while the fractal Lagrangean density remains invariant). Moreover, the fractal kink corresponds to a fractal pattern in the form of a Cooper type pair. We note that for motions on Peano curves at Compton scale, the above fractal pattern can be reduced to a standard Cooper type pair [14][15][16][17].

4. Topology at fractal scale and its implications

A fractal topological method can be applied because the admissible number of fractal kinks is determined by the fractal topological properties of the fractal symmetry group induced by equation (11) (details on the standard topological method can be found in [12]). In this context, the following problems must be solved: i) the number of admissible fractal kink solutions determined by the fractal topological properties of equation (11); ii) the fractal topological charge.

The fractal kink solution can be obtained as fractal mapping of a fractal spatial zero-sphere S^0, taken at infinity onto the fractal vacuum manifold of the model given by means of equation (11). The fractal homotopy group for this model is $\Pi_0(Z_0) = Z_2$ i.e. the model gives rise to two solutions: a constant solution g_F and the fractal kink solution. Details on a usual homotopy mapping are given in Ref. [12].

The fractal topological charge is:

$$q = \frac{1}{2}\int_{-\infty}^{\infty} j(\xi)d\xi = \frac{1}{2}\int_{-\infty}^{\infty} dg \tag{22}$$

The fractal vacuum solution (absence of spatial gradients) and the fractal kink solution can be characterized by attributing the $q = 0$ and $q = 1$, respectively. This result is obtained by an adequate normalization of g. Since equation (11) is a fractal Ginzburg – Landau type equation [13], it follows that $q = 0$ describes the fractal vacuum states, while $q=1$, by means of fractal kink solution, describes the fractal Cooper type pair [18][19][20].

Now, one can associate to these fractal topological charge values the fractal bit, that is a fractal system which can exist in two distinct fractal states (an unstructured state or of fractal vacuum and a structured one or of fractal Cooper type pair). These states are used in order to represent $0(dt)$ and $1(dt)$, that is a single binary fractal digit. In a particular case, for motions on Peano curves at Compton scale, the fractal bit is reduced to the quantum bit. Thus, the structural relations between the fractal Cooper type pairs generate a special topology, which implies defining the fractal bit.

5. Fractal cellular neural network

Since the general solution (with infinite "energy") of the GL type equation (11) has the explicit form,

$$g=\sqrt{\frac{2s^2}{1+s^2}}sn\left(\frac{\xi-\xi_0}{\sqrt{1+s^2}};s\right) \tag{23}$$

where sn is the elliptical Jacobi function of modulus s [21], the specific fractal potential becomes:

$$Q(\eta,s)=-\frac{1}{g}\frac{d^2g}{d\eta^2}=\left(1-g^2\right)=\frac{1-s^2}{1+s^2}+\frac{2s^2}{1+s^2}cn^2\left(\frac{\xi-\xi_0}{\sqrt{1+s^2}};s\right) \tag{24}$$

- see Figs. 1a-e. Therefore, the biological systems' structural units' dynamics can be described by cnoidal oscilation modes [22]. These modes, for $s=0$ or $s\to 0$ imply linear waves or wave packets,

$$Q(\eta,s=0,s\to 0)=\frac{1-s^2}{1+s^2}+\frac{2s^2}{1+s^2}\cos^2\left(\frac{\xi-\xi_0}{\sqrt{1+s^2}};s\right) \tag{25}$$

while for $s=1$ or $s\to 1$ they imply solitons or soliton packets,

$$Q(\eta,s=1,s\to 1)=\frac{1-s^2}{1+s^2}+\frac{2s^2}{1+s^2}\sec h^2\left(\frac{\xi-\xi_0}{\sqrt{1+s^2}};s\right). \tag{26}$$

The normalized fractal potential (24) takes a very simple expression which is directly proportional to the Cooper type pairs states density. When the Cooper type pairs states density, g^2, becomes zero, the fractal potential takes a finite value, $Q=1$. The fractal fluid is normal and there are no coherent structures (Cooper type pairs) in it. When g^2 becomes 1, the fractal potential is zero, i.e., the entire quantity of energy of the fractal fluid is transferred to its coherent structures, i.e., to the superconducting type pairs. Then the fractal fluid becomes "superconducting". Therefore, one can assume that the energy from the fractal fluid can be stoked by transforming all the environment's entities into coherent structures (Cooper type pairs) and then "freezing" them. The "superconducting" fluid acts as an energy accumulator through the fractal potential (24).

The cnoidal oscillation modes can be assimilated with a non-linear Toda lattice [23][24][25][26]. Now, by mapping these modes, a fractal cellular neural network can be defined. For details on this process. [13][27].

5. Conclusions

The main conclusions of the present papers are the following:

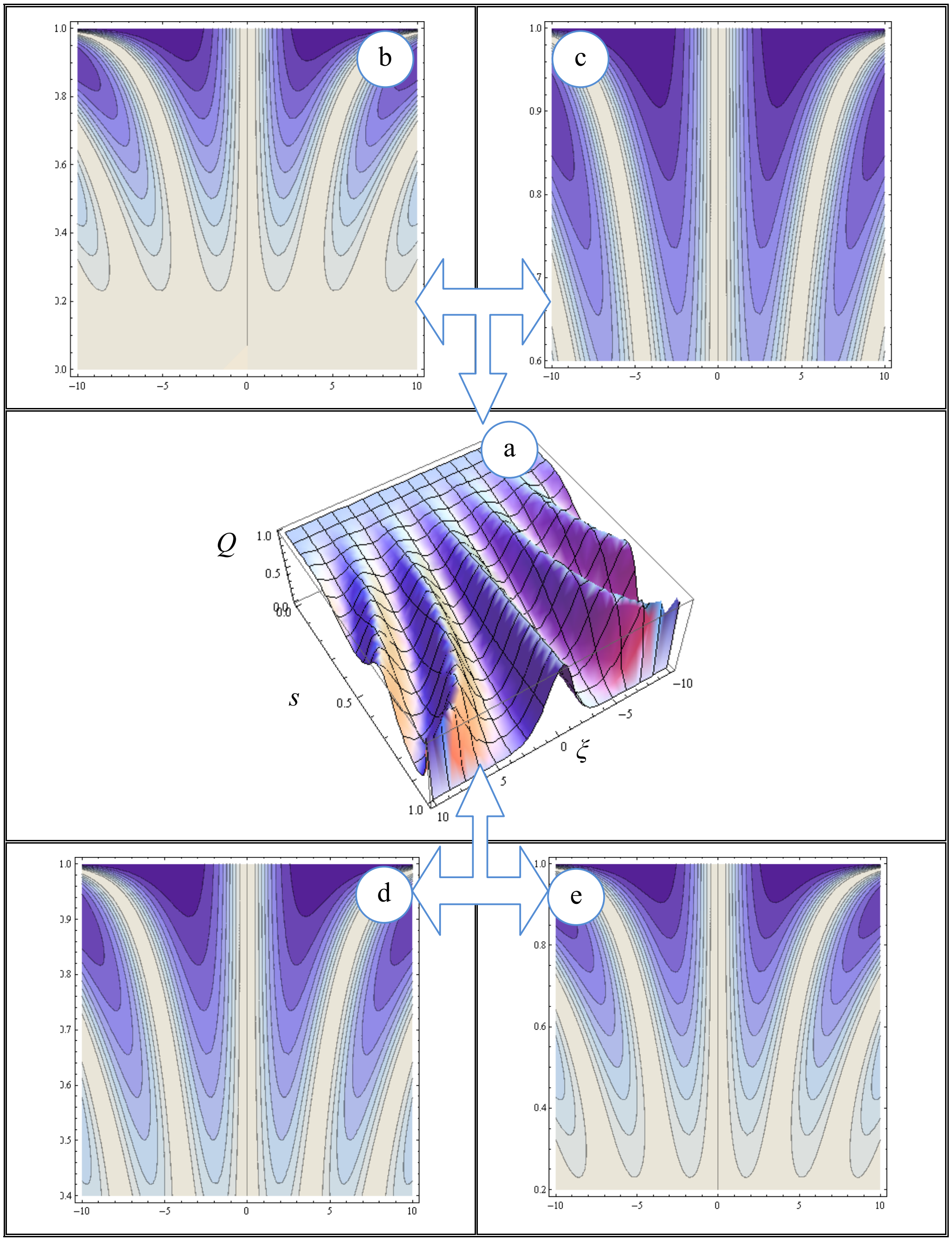
i) The hydrodynamic version of the scale relativity theory in arbitrary constant fractal dimension is presented (fractal hydrodynamics);

ii) Assuming that the external scalar potential is proportional with the fractal states density, the one-dimensional solution with finite fractal "energy" in the form of fractal kink is obtained. This solution breaks the fractal vacuum symmetry and generates fractal Cooper type pairs by means of fractal spontaneous symmetry breaking mechanisms. Then, the phase coherence of the fractal pairs will produce a self-structuring of the fractal vacuum which is interpreted as a tendency of the system to make structures (patterns) in the form of fractal Cooper type pairs. In such a manner, biological systems self-structuring can manifest;

iii) Since the admissible number of fractal kinks is determined by the fractal topological properties of the fractal symmetry group of equation (11), a topological fractal method can be applied. Then, the fractal bit and, in particularly, the quantum bit, are obtained;

iv) It can be shown that, by infinite energy solution mapping, a fractal cellular neural network can be defined;

v) The simultaneous presence in biological systems both of the "hardware" (cell, tissue, organ etc.) and of the "software" (fractal bit, fractal cellular neural networks etc.), denotes a higher class of evolution through external medium adjustment specific mechanisms. For example, compensatory growth is such a mechanism, of regenerative type, that can take place in a number of human organs after the organs are either damaged, removed, or cease to function [28]. Additionally, increased functional demand can also stimulate this growth in tissues and organs. In the case of the lung we observe the postpneumonectomy mechanism [3][4]. This concept is different from remodeling (capillary congestion, increasing in air content). The absolute increase in tissue after pneumonectomy and blood flow surfaces coupled with an enlargement of the conducting airways may provide a decrease in hypoxia. Postpneumonectomy compensation, which is slow for an adult lung, is more present in children. The cells expand by hyperplasia and/or hypertrophy.



Figs. 1a-e. 3D dependence of the specific fractal potential, Q on the non-linear degree, s and normalized coordinate, ξ (a); contour curves of the specific fractal potential for various non-linear degree (b-e)

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