

## APPLICATION OF DECOMPOSITION TECHNIQUE AND EFFICIENT METHODS FOR THE APPROXIMATE SOLUTION OF NONLINEAR EQUATIONS

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*In this paper, we suggest and analyze two new iterative methods for solving nonlinear equations using the system of coupled equations together with decomposition technique. Various numerical examples are given to illustrate the efficiency and performance of these iterative methods. These new iterative methods may be sighted as an addition and generalization of the existing methods for solving nonlinear equations.*

**Keywords:** System of coupled equations; Multi-step methods; Convergence; Numerical examples

### 1. Introduction

Several techniques are modified for finding the approximate solutions of the nonlinear equation  $f(x) = 0$ . Iterative methods are being developed by using techniques such as including Taylor series, quadrature formulas, homotopy and decomposition techniques. See [1-14] and the references therein.

Consider the nonlinear equation of the type

$$f(x) = 0. \quad (1)$$

We can rewrite (1), in the following equivalent form as:

$$x = h(x). \quad (2)$$

which is a fixed point problem. This alternative equivalent formulation plays an important and fundamental part in developing various iterative methods for solving nonlinear equation. We use the fixed point formulation (2) to suggest the following iterative methods.

For a given initial guess  $x_0$ , find the approximation solution  $x_{n+1}$  by the following iterative schemes:

$$x_{n+1} = h(x_n). \quad n = 1, 2, 3... \quad (3)$$

Such type of iterative methods are called the explicit method, see [13]. In a similar way, we can use the fixed point formulation (2) to suggest the following iterative method.

$$x_{n+1} = h(x_{n+1}). \quad n = 1, 2, 3... \quad (4)$$

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Methods formulated in the form described as (4) are called the implicit methods. We remark that, to execute these iterative methods, one generally uses the predictor-corrector technique. Some well known implicit and explicit type of iterative methods are known as Newton method, Halley method and Householder method [13] and presented in [8-12].

Newton's method and its modifications are being applied to locate the approximate solutions of nonlinear equations, [1–15] and the references therein. Abbasbandy [1] and Chun [2] have proposed and studied several one-step and two-step iterative methods with higher order convergence by using the Adomian decomposition technique. Abbasbandy [1] and Chun [2] used the higher order differential derivative for such purpose, which is the drawback of the technique. To overcome this drawback Noor [7, 9], Noor and Noor [6] and Noor et al. [8] considered another decomposition technique which does not involve the derivative of the Adomian polynomial.

In this paper, we use this alternative decomposition to construct some multi-step iterative methods for solving nonlinear equations. Results obtained in this paper convey that this new technique of decomposition is a promising tool and can be considered as a substitute of the Adomian decomposition method. In Section 2, we sketch the main ideas of this alternative decomposition technique and develop one-step, two-step and three-step iterative methods for solving nonlinear equations. Several numerical examples are given to illustrate the efficiency and the performance of the new iterative methods. Our results can be considered as an important improvement and refinement of the previously known results.

## 2. Technique and new suggested iterative methods

Assume that  $\alpha$  is a simple root of nonlinear equation (1) and  $\gamma$  is an initial guess sufficiently close to  $\alpha$ . We can rewrite the nonlinear equation (1) as a coupled system as:

$$f(\gamma) + (x - \gamma) \left[ \frac{f'(\gamma) + 3f'\left(\frac{2\gamma+x}{3}\right) + 3f'\left(\frac{\gamma+2x}{3}\right) + f'(x)}{8} \right] + g(x) = 0, \quad (5)$$

or

$$g(x) = f(x) - f(\gamma) - (x - \gamma) \left[ \frac{f'(\gamma) + 3f'\left(\frac{2\gamma+x}{3}\right) + 3f'\left(\frac{\gamma+2x}{3}\right) + f'(x)}{8} \right], \quad (6)$$

where  $\gamma$  is the initial approximation for a zero of (1). We can rewrite equation (5) in the following form:

$$x = \gamma - 8 \left[ \frac{f(\gamma) + g(x)}{f'(\gamma) + 3f'\left(\frac{2\gamma+x}{3}\right) + 3f'\left(\frac{\gamma+2x}{3}\right) + f'(x)} \right]. \quad (7)$$

We express (7), in the following form

$$x = c + \hbar N(x), \quad (8)$$

where

$$c = \gamma, \quad (9)$$

and

$$N(x) = -8 \left[ \frac{f(\gamma) + g(x)}{f'(\gamma) + 3f'\left(\frac{2\gamma+x}{3}\right) + 3f'\left(\frac{\gamma+2x}{3}\right) + f'(x)} \right]. \quad (10)$$

Here  $N(x)$  is a nonlinear function and  $\hbar \neq 0$  is an auxiliary parameter.

We now construct a sequence of higher order iterative methods for solving nonlinear equations by using the decomposition technique, which is mainly due to Daftardar-Gejji and Jafari [4]. This decomposition of the nonlinear function  $N(x)$  is quite different from that of Adomain decomposition.

The technique is designed to search for a solution having the series form

$$x = \sum_{i=0}^{\infty} x_i. \quad (11)$$

The nonlinear operator  $N$  can be decomposed as

$$N(x) = N(x_0) + \sum_{i=1}^{\infty} \left\{ N\left(\sum_{j=0}^i x_j\right) - N\left(\sum_{j=0}^{i-1} x_j\right) \right\}. \quad (12)$$

Combining (8), (11) and (12), we have

$$\sum_{i=0}^{\infty} x_i = c + \hbar N(x_0) + \hbar \sum_{i=1}^{\infty} \left\{ N\left(\sum_{j=0}^i x_j\right) - N\left(\sum_{j=0}^{i-1} x_j\right) \right\}. \quad (13)$$

Thus we have the following iterative scheme:

$$\begin{cases} x_0 = c, \\ x_1 = \hbar N(x_0), \\ x_2 = \hbar N(x_0 + x_1) - \hbar N(x_0), \\ \vdots \\ x_{m+1} = \hbar N\left(\sum_{j=0}^m x_j\right) - \hbar N\left(\sum_{j=0}^{m-1} x_j\right). \quad m = 1, 2, \dots \end{cases} \quad (14)$$

Then

$$x_1 + x_2 + \dots + x_{m+1} = \hbar N(x_0 + x_1 + \dots + x_m), \quad m = 1, 2, \dots \quad (15)$$

and

$$x = c + \sum_{i=1}^{\infty} x_i. \quad (16)$$

It can be shown that the series  $\sum_{i=0}^{\infty} x_i$  converges absolutely and uniformly to a unique solution of equation (8).

From (9) and (14), we get

$$x_0 = c = \gamma, \quad (17)$$

from (6), (10) and by using the idea of Yun [15], we obtain

$$g(x_0) = 0, \quad (18)$$

and

$$\begin{aligned} x_1 &= \hbar N(x_0) = -8\hbar \left[ \frac{f(\gamma) + g(x_0)}{f'(\gamma) + 3f'\left(\frac{2\gamma+x_0}{3}\right) + 3f'\left(\frac{\gamma+2x_0}{3}\right) + f'(x_0)} \right] \\ &= -8\hbar \left[ \frac{f(\gamma)}{f'(\gamma) + 3f'\left(\frac{2\gamma+x_0}{3}\right) + 3f'\left(\frac{\gamma+2x_0}{3}\right) + f'(x_0)} \right]. \end{aligned} \quad (19)$$

Note that  $x$  is approximated by

$$X_m = x_0 + x_1 + x_2 + \cdots + x_m, \quad (20)$$

where  $\lim_{m \rightarrow \infty} X_m = x$ .

For  $m = 0$ ,

$$x \approx X_0 = x_0 = c = \gamma. \quad (21)$$

For  $m = 1$ ,

$$\begin{aligned} x \approx X_1 &= x_0 + x_1 = \gamma - 8\hbar \left[ \frac{f(\gamma)}{f'(\gamma) + 3f'\left(\frac{2\gamma+x_0}{3}\right) + 3f'\left(\frac{\gamma+2x_0}{3}\right) + f'(x_0)} \right] \\ &= \gamma - \hbar \frac{f(\gamma)}{f'(\gamma)}. \end{aligned} \quad (22)$$

This formulation allows us to suggest the following one-step iterative method for solving the nonlinear equation (1).

**Algorithm 2.1.** For a given  $x_0$ , compute the approximate solution  $x_{n+1}$  by the following iterative scheme:

$$x_{n+1} = x_n - \hbar \frac{f(x_n)}{f'(x_n)}, \quad f'(x_n) \neq 0. \quad n = 0, 1, 2, \dots$$

For  $\hbar = 1$ , it is well known Newton's method [13] for solving nonlinear equations.

From (22), we get

$$x_0 + x_1 - \gamma = -\hbar \frac{f(\gamma)}{f'(\gamma)}, \quad (23)$$

from (6), (10) and by applying the suggestion of Yun [15], we have

$$\begin{aligned}
g(x_0 + x_1) &= f(x_0 + x_1) - f(\gamma) \\
&\quad - (x_0 + x_1 - \gamma) \left[ \frac{f'(\gamma) + 3f'\left(\frac{2\gamma + x_0 + x_1}{3}\right) + 3f'\left(\frac{\gamma + 2(x_0 + x_1)}{3}\right) + f'(x_0)}{8} \right] \\
&= f(x_0 + x_1) - f(\gamma) \\
&\quad + \hbar \frac{f(\gamma)}{8f'(\gamma)} \left[ f'(\gamma) + 3f'\left(\frac{2\gamma + x_0 + x_1}{3}\right) + 3f'\left(\frac{\gamma + 2(x_0 + x_1)}{3}\right) + f'(x_0) \right]. \tag{24}
\end{aligned}$$

and

$$\begin{aligned}
x_1 + x_2 = \hbar N(x_0 + x_1) &= -8\hbar \left[ \frac{f(\gamma) + g(x_0 + x_1)}{f'(\gamma) + 3f'\left(\frac{2\gamma + x_0 + x_1}{3}\right) + 3f'\left(\frac{\gamma + 2(x_0 + x_1)}{3}\right) + f'(x_0)} \right] \\
&= -\hbar^2 \frac{f(\gamma)}{f'(\gamma)} - \hbar \frac{8f(x_0 + x_1)}{f'(\gamma) + 3f'\left(\frac{2\gamma + x_0 + x_1}{3}\right) + 3f'\left(\frac{\gamma + 2(x_0 + x_1)}{3}\right) + f'(x_0)}. \tag{25}
\end{aligned}$$

For  $m = 2$ ,

$$\begin{aligned}
x &\approx X_2 = x_0 + x_1 + x_2 = c + \hbar N(x_0 + x_1) \\
&= \gamma - \hbar^2 \frac{f(\gamma)}{f'(\gamma)} - \hbar \frac{8f(x_0 + x_1)}{f'(\gamma) + 3f'\left(\frac{2\gamma + x_0 + x_1}{3}\right) + 3f'\left(\frac{\gamma + 2(x_0 + x_1)}{3}\right) + f'(x_0)}. \tag{26}
\end{aligned}$$

Using the above relation, we can suggest the following two-step iterative method for solving nonlinear equation (1).

**Algorithm 2.2.** For a given  $x_0$ , compute the approximate solution  $x_{n+1}$  by the iterative following scheme:

$$\begin{aligned}
y_n &= x_n - \hbar \frac{f(x_n)}{f'(x_n)}, \\
x_{n+1} &= x_n - \hbar^2 \frac{f(x_n)}{f'(x_n)} - \hbar \frac{8f(y_n)}{f'(x_n) + 3f'\left(\frac{2x_n + y_n}{3}\right) + 3f'\left(\frac{x_n + 2y_n}{3}\right) + f'(y_n)}.
\end{aligned}$$

Algorithm 2.2 is our newly derived iterative method in this paper for obtaining the approximate solution of nonlinear equations.

From (26), we obtain

$$x_0 + x_1 + x_2 - \gamma = -\hbar^2 \frac{f(\gamma)}{f'(\gamma)} - \hbar \frac{8f(x_0 + x_1)}{f'(\gamma) + 3f'\left(\frac{2\gamma + x_0 + x_1}{3}\right) + 3f'\left(\frac{\gamma + 2(x_0 + x_1)}{3}\right) + f'(x_0)}, \tag{27}$$

from (6), (10) and by using the idea of Yun [15], we get

$$\begin{aligned}
g(x_0 + x_1 + x_2) &= f(x_0 + x_1 + x_2) - f(\gamma) - (x_0 + x_1 + x_2 - \gamma) \\
&\times \left[ \frac{f'(\gamma) + 3f'\left(\frac{2\gamma + x_0 + x_1 + x_2}{3}\right) + 3f'\left(\frac{\gamma + 2(x_0 + x_1 + x_2)}{3}\right) + f'(x_0 + x_1 + x_2)}{8} \right], \\
&= f(x_0 + x_1 + x_2) - f(\gamma) \\
&\quad - \frac{1}{8} \left( -\hbar^2 \frac{f(\gamma)}{f'(\gamma)} - \hbar \frac{8f(x_0 + x_1)}{f'(\gamma) + 3f'\left(\frac{2\gamma + x_0 + x_1}{3}\right) + 3f'\left(\frac{\gamma + 2(x_0 + x_1)}{3}\right) + f'(x_0 + x_1)} \right) \\
&\quad \times \left[ f'(\gamma) + 3f'\left(\frac{2\gamma + x_0 + x_1 + x_2}{3}\right) + 3f'\left(\frac{\gamma + 2(x_0 + x_1 + x_2)}{3}\right) + f'(x_0 + x_1 + x_2) \right] \quad (28)
\end{aligned}$$

and

$$\begin{aligned}
x_1 + x_2 + x_3 &= \hbar N(x_0 + x_1 + x_2) = \\
&-8\hbar \left[ \frac{f(\gamma) + g(x_0 + x_1 + x_2)}{f'(\gamma) + 3f'\left(\frac{2\gamma + x_0 + x_1 + x_2}{3}\right) + 3f'\left(\frac{2\gamma + x_0 + x_1 + x_2}{3}\right) + f'(x_0 + x_1 + x_2)} \right] \\
&= -\hbar^3 \frac{f(\gamma)}{f'(\gamma)} - \hbar^2 \frac{8f(x_0 + x_1)}{f'(\gamma) + 3f'\left(\frac{2\gamma + x_0 + x_1}{3}\right) + 3f'\left(\frac{2\gamma + x_0 + x_1}{3}\right) + f'(x_0 + x_1)} \\
&\quad - \hbar \frac{8f(x_0 + x_1 + x_2)}{f'(\gamma) + 3f'\left(\frac{2\gamma + x_0 + x_1 + x_2}{3}\right) + 3f'\left(\frac{2\gamma + x_0 + x_1 + x_2}{3}\right) + f'(x_0 + x_1 + x_2)}. \quad (29)
\end{aligned}$$

For  $m = 3$ ,

$$\begin{aligned}
x &\approx X_3 = x_0 + x_1 + x_2 + x_3 = c + \hbar N(x_0 + x_1 + x_2) \\
&= \gamma - \hbar^3 \frac{f(\gamma)}{f'(\gamma)} - \hbar^2 \frac{8f(x_0 + x_1)}{f'(\gamma) + 3f'\left(\frac{2\gamma + x_0 + x_1}{3}\right) + 3f'\left(\frac{2\gamma + x_0 + x_1}{3}\right) + f'(x_0 + x_1)} \\
&\quad - \hbar \frac{8f(x_0 + x_1 + x_2)}{f'(\gamma) + 3f'\left(\frac{2\gamma + x_0 + x_1 + x_2}{3}\right) + 3f'\left(\frac{2\gamma + x_0 + x_1 + x_2}{3}\right) + f'(x_0 + x_1 + x_2)}. \quad (30)
\end{aligned}$$

Using this formulation, we can suggest the following three-step iterative method for solving nonlinear equation (1).

**Algorithm 2.3.** For a given  $x_0$ , compute the approximate solution  $x_{n+1}$  by the iterative following scheme.

$$y_n = x_n - \hbar \frac{f(x_n)}{f'(x_n)},$$

$$z_n = x_n - \hbar^2 \frac{f(x_n)}{f'(x_n)} - \hbar \frac{8f(y_n)}{f'(x_n) + 3f'\left(\frac{2x_n+y_n}{3}\right) + 3f'\left(\frac{x_n+2y_n}{3}\right) + f'(y_n)},$$

$$x_{n+1} = x_n - \hbar^3 \frac{f(x_n)}{f'(x_n)} - \hbar^2 \frac{8f(y_n)}{f'(x_n) + 3f'\left(\frac{2x_n+y_n}{3}\right) + 3f'\left(\frac{x_n+2y_n}{3}\right) + f'(y_n)}$$

$$- \hbar \frac{8f(z_n)}{f'(x_n) + 3f'\left(\frac{2x_n+z_n}{3}\right) + 3f'\left(\frac{x_n+2z_n}{3}\right) + f'(z_n)}.$$

Algorithm 2.3 is another newly derived method in this paper for solving nonlinear equations (1) and is one of the main motivations of this paper.

### 3. Convergence Analysis

In this section, we consider the convergence criteria of the iterative methods developed in section 2.

**Theorem 3.1.** Let  $\alpha \in I$  be a simple zero of sufficiently differentiable function  $f : I \subseteq \mathbb{R} \rightarrow \mathbb{R}$  for an open interval  $I$ . If  $x_0$  is sufficiently close to  $\alpha$ , then for  $\hbar=1$  the iterative method defined by Algorithm 2.3 has fourth-order convergence.

**Proof.** Let  $\alpha$  be a simple zero of  $f(x)$ . Then by expanding  $f(x_n)$  and  $f'(x_n)$  in Taylor's series about  $\alpha$ , we have

$$f(x_n) = f'(\alpha) \left[ e_n + c_2 e_n^2 + c_3 e_n^3 + c_4 e_n^4 + O(e_n^5) \right], \quad (31)$$

and

$$f'(x_n) = f'(\alpha) \left[ 1 + 2c_2 e_n + 3c_3 e_n^2 + 4c_4 e_n^3 + 5c_5 e_n^4 + O(e_n^5) \right], \quad (32)$$

where

$$c_k = \frac{1}{k!} \frac{f^{(k)}(\alpha)}{f'(\alpha)}, \quad k = 2, 3, \dots \text{ and } e_n = x_n - \alpha.$$

From (31) and (32), we have

$$y_n = \alpha + (1-\hbar)e_n + \hbar c_2 e_n^2 + 2(\hbar c_3 - \hbar c_2^2)e_n^3 + (-7\hbar c_2 c_3 + 3\hbar c_4 + 4\hbar c_2^3)e_n^4 + O(e_n^5). \quad (33)$$

Expanding  $f(y_n)$ ,  $f'(y_n)$ ,  $f'\left(\frac{2x_n+y_n}{3}\right)$  and  $f'\left(\frac{x_n+2y_n}{3}\right)$  in Taylor series about  $\alpha$  and using (33), we have after adding and simplifying

$$f'(x_n) + 3f'\left(\frac{2x_n+y_n}{3}\right) + 3f'\left(\frac{x_n+2y_n}{3}\right) + f'(y_n) =$$

$$f'(\alpha) \left[ (1-\hbar + \hbar^2 + \hbar^3)e_n + \frac{1}{3}(\hbar^6 - 2\hbar^5 - \hbar^4 + 2\hbar^3 - \hbar + 1)e_n^2 \right] + O(e_n^3). \quad (34)$$

Using (33) and (34), we get

$$\frac{8f(y_n)}{f'(x_n) + 3f'\left(\frac{2x_n+y_n}{3}\right) + 3f'\left(\frac{x_n+2y_n}{3}\right) + f'(y_n)} =$$

$$(1-\hbar)e_n + (2\hbar-1)e_n^2 + (5\hbar c_3 - 2c_3 - c_3\hbar^2 + \hbar^2 c_2^2 - 6\hbar c_2^2 + 2c_2^2)e_n^3 + O(e_n^4). \quad (35)$$

Equations (33) and (35) help to obtain the following

$$z_n = (\hbar - \hbar^2 + \hbar^3)e_n - \hbar c_2(1 - \hbar + 2\hbar^2)e_n^2$$

$$+ \hbar(\hbar^3 c_2^2 - \hbar^3 c_3 - 6\hbar^2 c_2^2 + 5\hbar^2 c_3 - 2c_3\hbar - 2c_2^2 + 2c_3 + 2\hbar c_2^2)e_n^3 + O(e_n^4). \quad (36)$$

By using (35), (36) and  $e_{n+1} = x_{n+1} - \alpha$ , we have

$$e_{n+1} = (1 - \hbar - \hbar^2 + \hbar^3)e_n + \hbar c_2(-3\hbar^2 + \hbar^3 + 1 + \hbar)e_n^2 + 2\hbar(2\hbar^3 c_3 - 3\hbar^3 c_2^2 + 5\hbar^2 c_2^2$$

$$- c_2^2 + c_3 + \hbar c_3 - 4\hbar^2 c_3 - \hbar c_2^2)e_n^3 + \hbar(27\hbar^3 c_3^2 + 45\hbar^2 c_2 c_3 - 7\hbar c_2 c_3 - \hbar^4 c_4 + 10\hbar^3 c_4$$

$$- 36\hbar^3 c_2 c_3 - 3\hbar c_4 + \hbar^5 c_4 - 4\hbar^4 c_2^3 - \hbar^5 c_2^3 - \hbar^6 c_2^3 - c_4 \hbar^6 + 4\hbar c_2^3 - 15\hbar^2 c_4 - 4\hbar^4 c_2^3$$

$$+ 4c_2^3 + 4\hbar c_2^3 - 3\hbar^4 c_2 c_3 - 2\hbar^6 c_2 c_3 - 4\hbar c_2^3 + 30\hbar^2 c_2^3)e_n^4 + O(e_n^5). \quad (37)$$

For  $\hbar = 1$ , we obtain from (37), the following error equation

$$e_{n+1} = c_2^3 e_n^4 + O(e_n^5). \quad (38)$$

Error equation (38) shows that for  $\hbar = 1$  the Algorithm 2.3 has fourth-order convergence.  $\square$

In a similar way, one can observe the analysis of convergence of Algorithm 2.2.

#### 4. Numerical Results

We now present some examples to demonstrate the effectiveness of the newly developed two-step and three-step iterative methods in this paper. We compare the Newton's method (NM), the method of Hasanov et al. [5] (HM), the method of Chun [2] (CM), Algorithm 2.2 (NR1), and the Algorithm 2.3 (NR2) introduced in this paper. We used  $\hbar = 1$  and  $\varepsilon = 10^{-15}$ . The following stopping criteria is used for computer programs:

$$(i). |x_{n+1} - x_n| < \varepsilon, \quad (ii). |f(x_n)| < \varepsilon.$$

The computational order of convergence  $p$  is approximated also (See [12]).

We consider the following nonlinear equations as test problems which are same as Noor and Noor [6].

$$f_1(x) = \sin^2 x - x^2 + 1, \quad f_2(x) = x^2 - e^x - 3x + 2, \quad f_3(x) = (x-1)^3 - 1,$$

$$f_4(x) = x^3 - 10, \quad f_5(x) = xe^{x^2} - \sin^2 x + 3\cos x + 5, \quad f_6(x) = e^{x^2+7x-30} - 1.$$



Table 4.1

	IT	$x_n$	$f(x_n)$	$\delta$	$p$
$f_1, x_0 = -1$					
NM	7	1.404491648215341226	-1.04e-50	7.33e-26	2.00003
HM	5	1.404491648215341226	0	6.98e-32	3.03470
CM	5	1.404491648215341226	0	1.31e-17	2.85844
NR1	5	1.404491648215341226	0	7.31e-32	3.03479
NR2	4	1.404491648215341226	0	2.95e-25	4.33335
$f_2, x_0 = 2$					
NM	6	0.25753028543986076	2.93e-55	9.10e-28	2.00050
HM	4	0.25753028543986076	-3.91e-53	1.06e-17	3.49362
CM	4	0.25753028543986076	0	9.46e-29	4.57143
NR1	4	0.25753028543986076	-3.38e-53	1.01e-17	3.48939
NR2	3	0.25753028543986076	0	1.08e-45	4.14860
$f_3, x_0 = 3.5$					
NM	8	2.00000000000000000000	2.06e-42	8.28e-22	2.00025
HM	6	2.00000000000000000000	0	1.10e-40	2.99061
CM	5	2.00000000000000000000	0	2.74e-24	3.53144
NR1	4	2.00000000000000000000	0	1.10e-40	2.99061
NR2	3	2.00000000000000000000	0	9.78e-43	3.86663
$f_4, x_0 = 1.5$					
NM	7	2.15443469003188372	2.06e-54	5.64e-28	2.00003
HM	5	2.15443469003188372	1.00e-58	4.57e-35	3.01855
CM	5	2.15443469003188372	1.00e-58	1.57e-22	3.48932
NR1	3	2.15443469003188372	-8.00e-59	4.57e-35	3.01855
NR2	3	2.15443469003188372	1.00e-58	9.15e-28	4.22908
$f_5, x_0 = -2$					
NM	9	-1.2076478271309189	-2.27e-40	2.73e-21	2.00085
HM	6	-1.2076478271309189	-2.38e-57	3.68e-20	3.00846
CM	6	-1.2076478271309189	-1.10e-58	2.15e-36	3.88967
NR1	5	-1.2076478271309189	-2.14e-57	3.56e-20	3.00831
NR2	4	-1.2076478271309189	8.00e-59	4.39e-21	4.01292
$f_6, x_0 = 3.5$					
NM	13	3.00000000000000000000	1.52e-47	4.21e-25	2.00023
HM	9	3.00000000000000000000	0	6.57e-39	2.99432
CM	8	3.00000000000000000000	0	2.12e-23	3.68024
NR1	7	3.00000000000000000000	0	6.03e-39	2.99434
NR2	6	3.00000000000000000000	0	1.31e-24	3.83853

## 5. Conclusion

In this paper, we have studied one-step, two-step and three-step numerical methods for solving nonlinear equations by using a different composition technique. Our method of derivation of the iterative methods is very simple as compared with the Adomian decomposition technique. This is another feature of the simplicity. Using the technique and idea of this paper, one can suggest and analyze higher-order multi-step iterative methods for solving nonlinear equations as well as system of nonlinear equations.

## Acknowledgement

Authors are grateful to the referees and the editor for their constructive and valuable comments, which influenced the final version of this paper. Authors are thankful to Dr. S. M. Junaid Zaidi (H.I., S.I.), Rector, COMSATS Institute of Information Technology, Pakistan for providing excellent research and academic environment.

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