A PROCEDURE TO OBTAIN THE PROBABILISTIC KITAGAWA-TAKAHASHI DIAGRAM

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An alternative way of interpreting the Kitagawa-Takahashi diagram for structural components is proposed. With this aim, the equivalent initial flaw size (EIFS) model, as a way of defining the initial defects of the structural components is used in conjunction with the probabilistic S-N model proposed by Castillo and Canteli, thus allowing the probabilistic distribution of the EIFS to be generated and, consequently, a probabilistic definition of the KT diagram (P-KT) to be achieved. The proposed approach is applied to a notched plate made of P355NL1 steel, the results of predictions are analyzed and the deviations discussed.

Keywords: Kitagawa-Takahashi Diagram; Fatigue; Probabilistic model; Fracture Mechanics.

1. Introduction

The *Kitagawa-Takahashi* (*KT*) diagram [1] represents a boundary in terms of crack size and stress range for which infinite fatigue lifetime of structural and mechanical components can be safely ensured due to non-propagating micro- and macrocracks [2]. Generally, only a deterministic conception of the K-T diagram is considered in the practice despite the obvious necessity of incorporating its probabilistic dimension in a more reliable structural integrity design. Some exceptions are found in the literature. As the preliminary attempt, Fernández-Canteli et al. [3] incorporates the probabilistic information of the experimental S-N field, as provided by the P-S-N model proposed by Castillo and Fernández-Canteli [4], which relates crack size and lifetime, into the K-T diagram. On its turn, the probabilistic model of Pessard et al. [5] is based on the consideration of

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Weibull distributions for the two damage mechanisms: initiation (safe-life concept) and propagation (damage tolerance concept).

In this paper, an alternative way of promoting a probabilistic concept of the K-T diagram is intended using the equivalent initial flaw size (*EIFS)* concept based on fracture mechanics, particularly on the elastoplastic cyclic J-integral, in which the initial defects of the structural components are taken into account [6,7]. The inverse analysis proposed by Alves et al. [8] is applied to estimate the *EIFS* parameter and after considering the probabilistic *P-S-N* model developed by Castillo and Canteli [4], the probabilistic *Kitagawa-Takahashi* diagram (*P-KT*) is obtained for structural components. The proposed approach is applied to lifetime prediction of a notched plate made of P355NL1 steel, and the results analyzed and discussed.

2. Probabilistic *S-N* model by Castillo and Canteli

Castillo and Fernández-Canteli [4] derived a Weibull regression model for constant stress range and given stress level (e.g. stress ratio, mean stress). This model, being formulated in the stress space, is recommended for medium to high, or even very high cycle fatigue life prediction. The derivation of the model is based on the fulfilment of physical conditions (identification of the involved variables and dimensional analysis) and statistical requirements (weakest link principle, stability, limited range, limit behaviour). In addition, the fulfilment of the necessary compatibility condition between lifetime distribution, for given stress range, and the stress range distribution, for given lifetime, leads to a functional equation, the solution of which provides the following Weibull distribution, defining the probabilistic *S-N* field [4]:

$$F(logN; log\Delta\sigma) = p = 1 - exp\left\{-\left[\frac{(logN-B)(log\Delta\sigma-C)-\lambda}{\delta}\right]^{\beta}\right\};\ (logN - B)(log\Delta\sigma - C) \geq \lambda \quad (1)$$

where: N is the lifetime; $\Delta\sigma$ is the stress range; $F()$ is the Weibull cumulative distribution function (CDF) of N for given $\Delta\sigma$; $B=log(N_0)$, N_0 being a threshold value of lifetime; $C=log(\Delta\sigma_0)$, $\Delta\sigma_0$ being the endurance fatigue limit; and λ, β and δ are, respectively, the shape, scale and location Weibull model parameters, the latter defining the position of the zero-percentile curve. The model, as presented in Figure 1a) and defined by Equation (1), has been studied and successfully applied to different lifetime assessments [4,9,10] and extended to the case of variable stress level [11].

The normalized variable $V=(logN-B)(log\Delta\sigma-C)$ by simultaneous melting of the stress/strain ranges or amplitudes and the number of cycles allows the *S-N* field to be reduced to a simple Weibull CDF, see Figure 2, which can be occasionally relaxed to a Gumbel distribution. For a fixed stress range, the probability of failure increases monotonically with the number of cycles; in the same way, the probability of failure goes up by increasing stress ranges for a fixed number of cycles. By considering the normalized variable V, equivalent loading conditions are established, as those leading to the same probability of failure thus allowing a damage cumulative conversion to be formulated. Considering the *S-N* field of Figure 1b), the loading condition ($\Delta\sigma_A$, N_A) is equivalent to ($\Delta\sigma_B$, N_B) since they exhibit the same probability of failure as a result of showing the same normalizing variable, $V_A=V_B$:

$$V_A = V_B = (logN_A - B)(log\Delta\sigma_A - C) = (logN_B - B)(log\Delta\sigma_B - C) \quad (2)$$

$$\Delta\sigma_B = \exp\left[\frac{(logN_A - B)(log\Delta\sigma_A - C)}{(logN_B - B)} + C\right] = \exp\left[\frac{V_A}{(logN_B - B)} + C\right] \quad (3)$$

$$N_B = \exp\left[\frac{(logN_A - B)(log\Delta\sigma_A - C)}{(log\Delta\sigma_B - C)} + B\right] = \exp\left[\frac{V_A}{(log\Delta\sigma_B - C)} + B\right] \quad (4)$$

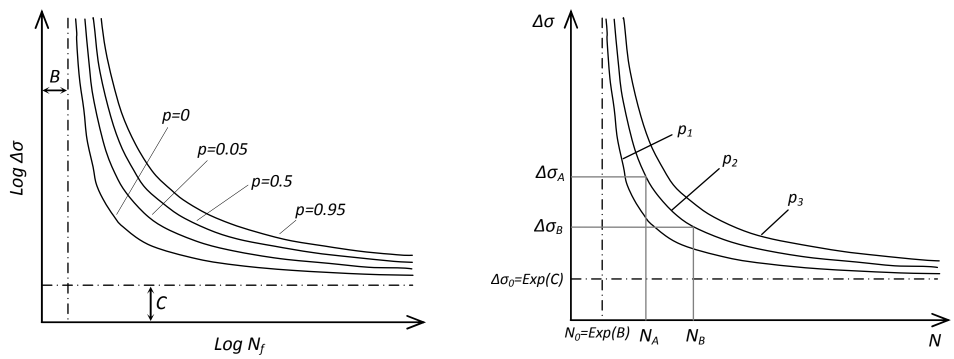


Fig. 1. Probabilistic *S-N* field: a) *S-N* model proposed by Castillo and Fernández-Canteli [4]; b) Representation of two equivalent loading conditions (same probability of failure and damage).

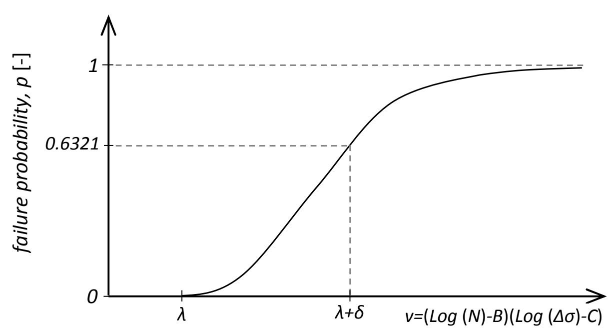


Fig. 2. Cumulative distribution function of the Weibull normalized variable V.

3. Equivalent Initial Flaw Size (EIFS) concept based on Fracture Mechanics and Cyclic J-Integral

The fatigue life evaluation based on the fracture mechanics approach for notched details is supported by material crack propagation laws [12]. The cyclic J-integral can be used to take into account the elastic–plastic deformations in the crack-tip area by means of the expression:

$$\frac{da}{dN} = f(\Delta J) \tag{5}$$

where *da/dN* is the fatigue crack growth rate, *ΔJ* is the range of the cyclic J-integral and *f*() is a function of the J-integral.

The number of cycles to failure may be computed by integrating the crack propagation law between the initial crack size, a_i and the final crack size, a_f:

$$N = \int_{a_i}^{a_f} \frac{da}{f(\Delta J)} \tag{6}$$

The material is assumed to exhibit surface defects acting as initial cracks. In order to allow the computation of the global fatigue life of the component to be performed, the initial crack size a_i, is supposed to be a material characteristic representing the *EIFS* of the material.

To consider the crack propagation regime I, an extension of the Paris-type crack growth law [12] is proposed by Alves et al. [8]:

$$\frac{da}{dN} = (\Delta J - \Delta J_{th}), \Delta J \geq \Delta J_{th} \tag{7}$$

A numerical integration of the propagation law based on cyclic J-integral is adopted by the following approximation:

$$N = \sum_{a_i}^{a_f} \frac{da}{f(\Delta J)} \tag{8}$$

where the equivalent initial flaw size (*EIFS*) is estimated by means of the inverse (back-extrapolation) analysis proposed by Alves et al. [8], the procedure of which is schematically depicted in Figure 3.

4. Probabilistic procedure applied to the Kitagawa-Takahashi diagram

The process proposed to obtain a probabilistic *Kitagawa-Takahashi* Diagram (*P-KT*) for notched structural details can be summarized as follows:

i) A probabilistic *S-N* field must be derived for the material or mechanical/structural component under consideration from stress-based fatigue data, using the probabilistic model by Castillo and Canteli [4].

ii) The equivalent initial flaw size (*EIFS*) is obtained for any probability of failure, using the fatigue crack propagation data and the *S-N* field represented by the percentile curves previously established in step i). The cyclic J-Integral is thereby adopted.

iii) Next, the relation between the variable *V=[log(N)-B][log(Δσ)-C]* vs. probability *p* is deduced, then the relation between *EIFS* vs. the normalized variable *V* is established from which, finally, the relation between *EIFS* vs. *p* can be found.

iv) As a last step, the probabilistic *Kitagawa-Takahashi* Diagram is derived. This procedure allows us to obtain an equivalent initial flaw size distribution (*EIFS-CDF*).

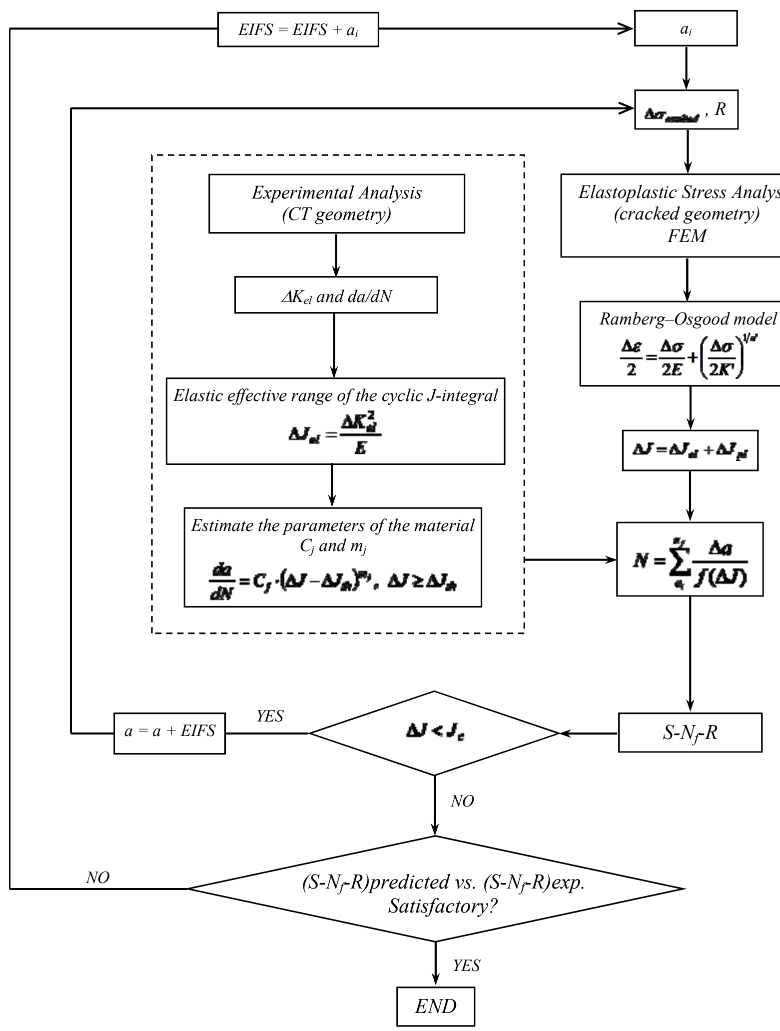


Fig. 3. Procedure proposed by Alves et al. [8] to estimate the *EIFS* using inverse analysis.

The schematic procedure for the derivation of the probabilistic Kitagawa-Takahashi diagram, as proposed in this paper, is represented in Figure 4.

The consideration of the normalized variable, *V*, allows the equivalence between two loading states to be established based on percentile (iso-probability) curves - these being interpreted as iso-initial flaw size (iso-*EIFS*) curves - and the probability of failure, *p*. In this way, the normalizing variable, *V*, may be understood as a possible alternative to *EIFS* measurements [9]. The consideration of the probability of failure associated to the equivalent initial flaw size (*EIFS*) parameter could be used advantageously for design purposes, namely to establish safety margins. In this sense, the percentile curves can be interpreted as representing different initial flaw sizes.

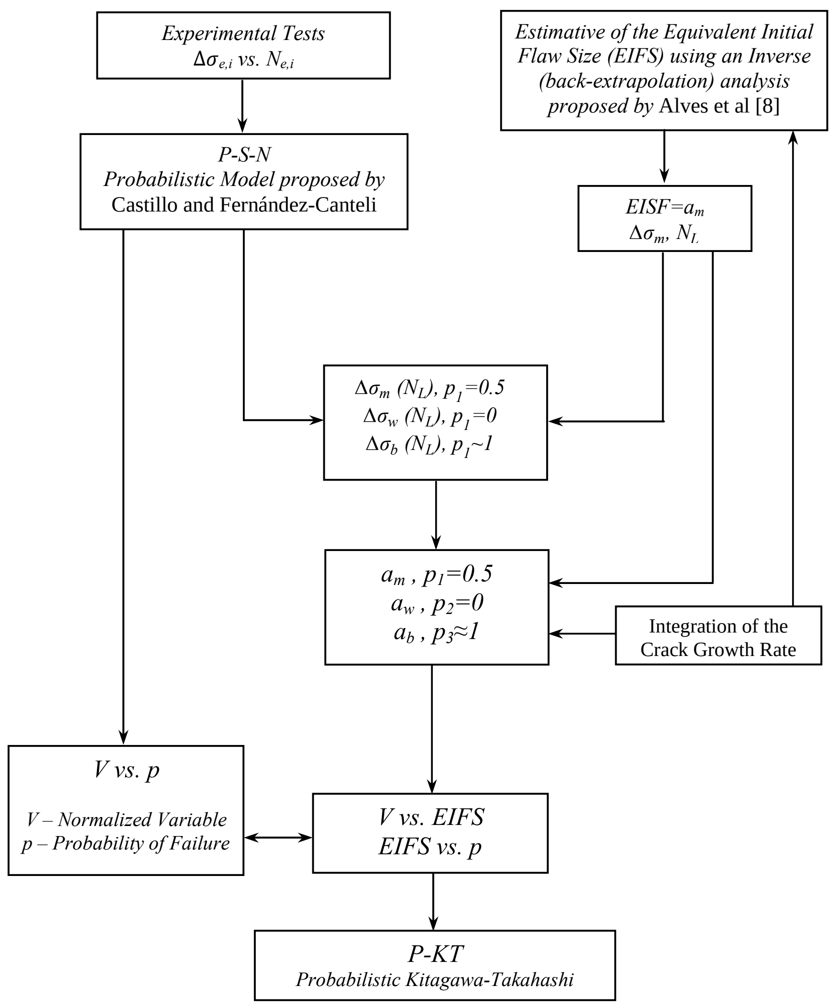


Fig. 4. Procedure adopted to compute the probabilistic Kitagawa-Takahashi diagram (P-KT).

The *S-N* curves are generally related to a unique probability of failure, $p=0.5$ or $p=0.05$, though the scatter of experimental data requires the definition of the whole *S-N* field as percentile curves based on statistical principles.

The percentile curves can be assumed to be associated with the probability of existence of a crack being less than a certain initial crack size, a_i, initially unknown (see Fig. 5).

The fatigue failure is governed by the maximum crack size, present in the specimen being tested, so that percentile curves with increasing probabilities of failure are related to diminishing crack sizes: the percentile curve $p=0$, corresponding to the greatest, or worst, among the maximum crack sizes of the population, i.e., $a_{i,w}$=max(a_{max}), which is denoted max-max crack size [3].

Similarly, the upper percentile curve, $p=1$, corresponds to the minimum, or best, of the maximum crack sizes of the population, i.e., $a_{i,b}$=min(a_{max}), which is denoted min-max crack size [3].

For practical purposes, the definition of the latter can be relaxed identifying $a_{i,b}$, as an initial crack size related to a high probability of failure, for instance, $p_b=0.90$ or 0.95. The two curves associated with $a_{i,w}$ and $a_{i,b}$ represent the two limiting sizes of the initial maximum defect corresponding to the particular surface finishing of the material tested [3].

For a given number of cycles to failure N_L, two different stress ranges $\Delta\sigma_b$ and $\Delta\sigma_w$ ($\Delta\sigma_b > \Delta\sigma_w$), are identified with the best and worst surface defects respectively [5].

For finite reference lifetime N_L, an engineering threshold value $\Delta J_{th,eng}$ can be found, particularly for the defect sizes $a_{i,w}$ and $a_{i,b}$, but also for a generic crack size $a_{i,m}$ ($a_{i,w} > a_{i,m} > a_{i,b}$) [3].

For $N_L \rightarrow \infty$, the $\Delta J_{th,eng}$ becomes the true threshold value ΔJ_{th} of the crack growth rate curve.

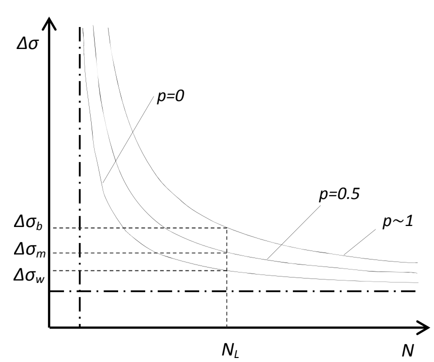


Fig. 5. Statistical principles applied to the reference lifetime, using the probabilistic *S-N* model proposed by Castillo and Fernández-Canteli [4].

5. Application to a notched plate made of P355NL1 steel

The proposed probabilistic procedure was applied to the experimental data derived from 5 mm thick plates made of P355NL1 steel [13,14]. Table 1 summarizes the mechanical properties of this steel. Figure 6 presents the crack propagation data for the P355NL1 steel according to the recommendations of the ASTM E647 standard. Experimental tension fatigue tests were carried out on notched plates, as those depicted in Figure 7, for a stress ratio R=0, the results of which are shown in Figure 8. The fatigue propagation law for stress R=0, used in this investigation, is presented in Figure 9 [8].

Table 1

Mechanical properties of the P355NL1 steel

Ultimate tensile strength, σ_{UTS} [MPa]	568
Monotonic yield strength, σ_y [MPa]	418
Young's modulus, E [GPa]	205.2
Poisson's ratio, ν	0.275
Cyclic hardening coefficient, K' [MPa]	777
Cyclic hardening exponent, n'	0.1068
Fatigue strength coefficient, σ'_f [MPa]	840.5
Fatigue strength exponent, b	-0.0808
Fatigue ductility coefficient, ε'_f	0.3034
Fatigue ductility exponent, c	-0.6016

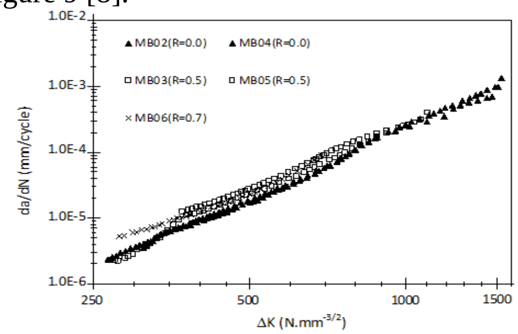


Fig. 6. Crack propagation data for the P355NL1 steel.

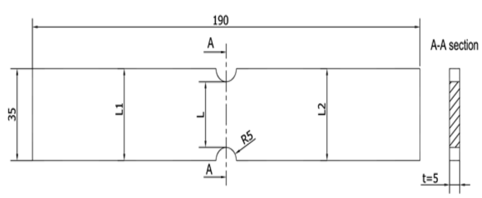


Fig. 7. Notched rectangular plate used in the tests (dimensions in mm).

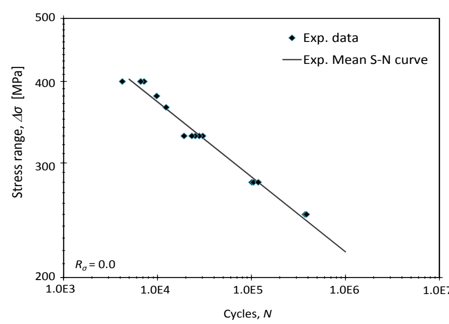


Fig. 8. S-N fatigue data from notched plates.

In order to apply the procedure proposed by Alves et al. [8] an elastoplastic stress analysis was performed for computation of the cyclic J-integral range at the notched structural detail using a finite element model. Figure 10 represents the results of the cyclic J-Integral as a function of the nominal stress range for a crack length, a=0.625mm, obtained for the notched plate [8]. The proposed procedure requires the probabilistic *S-N* field for the structural detail allowing to obtain *V vs. p* and in turn *V vs. EIFS*. Finally, the results of the proposed procedure are presented in Figure 11 allowing an equivalent initial flaw size distribution (EIFS-CDF) to be estimated.

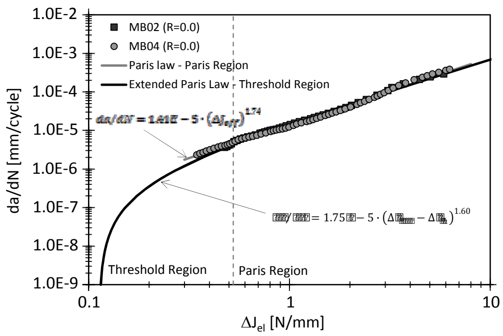


Fig. 9. Crack growth law adopted in this study.

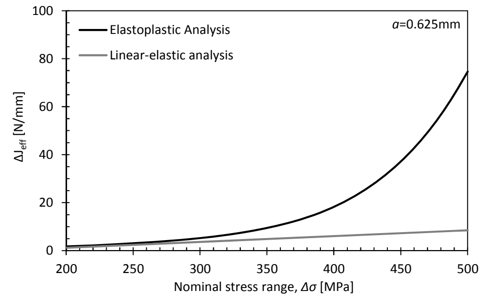
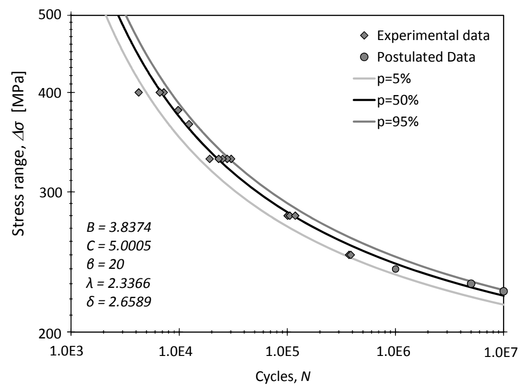
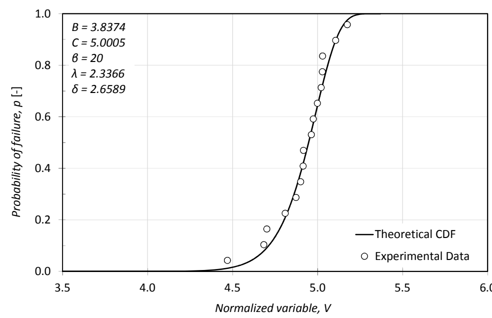


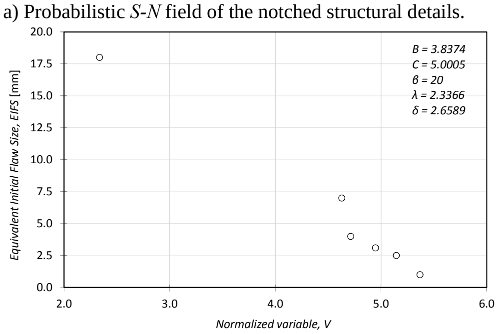
Fig. 10. Value of the cyclic J-Integral as a function of nominal stress range for the notched plate (a=0.625mm).



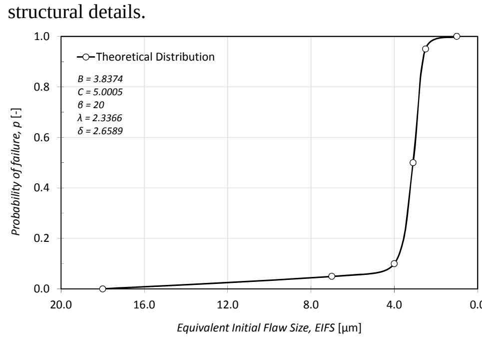
a) Probabilistic *S-N* field of the notched structural details.



b) Comparison between the experimental and theoretical cumulative distribution functions of the normalized variable *V* associated to the *S-N* data of the notched structural details.



c) Normalized variable *V* versus *EIFS* parameter.



d) Failure probability computed for the *EIFS* parameter.

Fig. 11. Application and results of the proposed probabilistic procedure.

6. Conclusions

The main conclusions derived from this study are the following:

- A procedure is proposed to obtain a probabilistic Kitagawa-Takahashi diagram (*P-KT*) providing higher reliability in practical design cases.
- The approach allows us to establish a connection between the probabilistic *S-N* field proposed by Castillo and Canteli [4], and the

equivalent initial flaw size (EIFS) concept based on fracture mechanics and cyclic J-integral. The approach can be applied indistinctly for both finite and infinite limit number of cycles.

- Further study is needed to allow an extension of the proposed approach to define the *KT* diagram for small cracks. i.e., in the low-cyclic fatigue (*LCF*) region, as well as the consideration of stochastic cracks growth rate curves, particularly in the threshold regime.

REFERENCES

[1]. *H. Kitagawa*, *S. Takahashi*. Applicability of fracture mechanics to very small cracks or cracks in the early stage. Proc. of the 2nd Int. Conf. on Mech. Behaviour of Materials, ASM, 627–631, 1976.

[2]. *M. Ciavarella , F. Monno.* On the possible generalizations of the Kitagawa–Takahashi diagram and of the El Haddad equation to finite life. Int. J. of Fatigue 2006; 28, pp. 1826-1837.

[3]. *A. Fernández-Canteli*, *R. Brighenti*, *E. Castillo*. Towards a probabilistic concept of the Kitagawa-Takahashi diagram. Proceedings of the 4th Int. Conference on Crack Paths (CP 2012), Gaeta, Italy, 2012, 1041-1048.

[4]. *E. Castillo*, *A. Fernández-Canteli*. A general regression model for lifetime evaluation and prediction, Int. J. Fract., 2001; 107, pp. 117–137.

[5] *E. Pessard, D. Bellett, F. Morel, I. Koutiri.* A mechanistic approach to the Kitagawa–Takahashi diagram using a multiaxial probabilistic framework, Eng.Fract. Mechanics 2013; 109, pp. 89-104.

[6]. *Y. Xiang*, *Z. Lu*, *Y. Liu*. Crack growth-based fatigue life prediction using an equivalent initial flaw model. Part I: Uniaxial loading. Int. J. of Fatigue, Vol. 32, Num. 2, pp. 341-349, 2010.

[7]. *G. Savaidis*, *A. Savaidis*, *P. Zerres*, *M. Vormwald*. Mode I fatigue crack growth at notches considering crack closure. International Journal of Fatigue, Vol. 32, Num. 10, pp. 1543-1558, 2010.

[8]. *A.S.F. Alves*, *L.M.C.M.V. Sampayo*, *J.A.F.O. Correia*, *A.M.P. De Jesus*, *P.M.G.P. Moreira*, *P.J.S. Tavares*. Fatigue life prediction based on crack growth analysis using an equivalent initial flaw size model: Application to a notched geometry. Procedia Engineering 114 (2015) 730–737.

[9]. *E. Castillo*, *M. López-Aenlle*, *A. Ramos*, *A. Fernández-Canteli*, *R. Kieselbach*, *V. Esslinger*. Specimen length effect on parameter estimation in modelling fatigue strength by Weibull distribution. Int. J. Fatigue, 2006; 28, pp. 1047–1058.

[10]. *E. Castillo*, *A. Fernández-Canteli*, *M.L. Ruiz-Ripoll*. A general model for fatigue damage due to any stress history. Int. J. Fatigue, 2008; 30, pp. 150–164.

[11]. *E. Castillo*, *A. Fernández-Canteli*. A unified statistical methodology for modeling fatigue damage. Springer, 2009.

[12]. *P.C. Paris*, *F. Erdogan*. A critical analysis of crack propagation laws. Transactions of The ASME. Series E: Journal of Basic Engineering, Vol. 85, pp. 528-534, 1963.

[13]. *H.F.S.G. Pereira*, *A.M.P. de Jesus*, *A.S. Ribeiro*, *A.A. Fernandes*. Fatigue damage behavior of a structural component made of P355NL1 steel under block loading. Journal of Pressure Vessel Technology, Vol. 131, Num. 2, pp. 021407 (9 pages), 2009.

[14]. *J.A.F.O. Correia*, *A.M.P. De Jesus*, *A. Fernández-Canteli*. Local unified probabilistic model for fatigue crack initiation and propagation: Application to a notched geometry. Engineering Structures, Volume 52, July 2013, Pages 394-407.