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STARK SHIFT AND OSCILLATOR STRENGTHS IN A GaAs QUANTUM RING WITH OFF-CENTER DONOR IMPURITY

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We present a theoretical study of the lateral electric field effect on the Stark-shift, the polarizability and oscillator strength associated to off-center donor impurities in a GaAs disc-shaped quantum ring. Calculations are performed in the effective mass approximation and using a finite difference method. We found that in the range of the moderate field values, used in our computation, the Stark-shift exhibits a linear dependence, while the impurity polarizability tends rapidly tends to the saturation value. Moreover, the behavior of the oscillator strength associated with impurity interlevel transitions shows a strong dependence on the incident light polarization and impurity position within the structure. Our results suggest the new possibility of tuning the performance of impurity doped quantum rings using lateral electric fields.

Keywords: disc shape quantum dot, Stark effect, polarizability, oscillator strength

1. Introduction

Recent progress in nanofabrication technology allows the possibility of realizing quantum dots (QDs) with controllable size, shape and composition [1–3]. As consequence of quantum confinement effect, formation of discrete energy levels in QDs leads to specific electronic and optical properties, different than their bulk counterparts.

It is well-known that external perturbations give rise to remarkable changes in the electronic and optical response of quantum dots [4-7]. In this respect, understanding the interaction between the electric applied fields with carriers in low-dimensional semiconductor structures facilitates the opportunity for manipulation and control the physical properties [8-10]. The changes in the optical characteristics of semiconductor nanostructures in response to an electric field manifests itself in spectral shifts and changes in the intensities of the absorption maxima [11]. Moreover, semiconductor optoelectronic modulators based on the quantum-confined Stark effect (QCSE) are attractive transmitters to convert signals from the electronic into the optical domain and vice versa [12].

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The incorporation of impurities into a quantum dot affects considerably the optical and transport properties and can explain the new photo-luminescent transition in optoelectronic devices. In this context, understanding the impurity effects on the electronic states is important from both fundamental physics and device applications perspectives. Terzis and Baskoutas [13] investigated the effect of electric and magnetic field on the binding energy of an impurity donor in GaAs QD and they found that the binding energy decreases as the electric field increases. Morales *et al*. [14] included the effect of the hydrostatic pressure in their study, demonstrating that the impurity binding energy and polarizability can be tuned by means of an applied external electric field or by hydrostatic pressure. Using the variational method, Aderrasa *et al*. [15] calculated the Stark effect and the polarizability of shallow-donor impurity located in the center of lens shaped quantum dot.

Among the semiconductor nanostructures, quantum rings (QRs) are of special interest. It is already known that in these systems, in-plane electric fields may drastically change the electronic spectra, whereas the perpendicular electric fields have negligible effects since they do not affect the axial symmetry of the structure [16]. In this respect, in-plane applied electric fields in QR structures were shown to break the ring topology, to couple different angular momenta and to affect the inter-band oscillator strength [17-21].

Previously, we reported some effects of an off-center donor impurity on the nonlinear optical absorption, second and third harmonic generation in a GaAs two-dimensional disc-shaped QD under magnetic fields [22] and on the occurrence of the electromagnetically induced transparency phenomenon in the structure [23]. In this paper, we study the electric field effects on the Stark-shift of impurity ground state in a two-dimensional quantum ring represented by a pseudoharmonic potential. The energy spectra and wave functions are obtained within the effective mass approximation by using a finite element method [24]. The paper is organized as follows: In Section 2, the theoretical framework is briefly given. Numerical results presented in Section 3 are followed by the conclusions given in Section 4.

2. Theory

We consider a conduction band electron confined into a QR with $R_1\,(R_2)$ the inner (outer) in-plane radius and height L_z, under the simultaneous action of the Coulomb attraction due to an ionized donor and of a lateral applied electric field *F*. For a flat QR, for which the condition $R_2 - R_1 >> L_z$ is fulfilled, we can use the adiabatic approximation and decouple the motion along the *z*-axis from the *xy* one. In this case, the main features of the spectrum are essentially determined by the confinement in the *xy*-plane [25].

In the framework of the effective mass approximation and choosing the electric field direction as *x*-axis, the single-particle Hamiltonian is

$$H = H_0 + eFx - \frac{e^2}{4\pi\varepsilon_0\varepsilon_r\left|\vec{r}-\vec{r}_i\right|} \tag{1}$$

where

$$H_0 = -\frac{\hbar^2}{2m^*}\Delta + V(\vec{r}). \tag{2}$$

Here $\vec{r} = (x, y)$ is the in-plane vector position for the electron, ε_0 the vacuum dielectric permittivity, ε_r the static dielectric constant of the ring material, $\vec{r}_i = (x_i, y_i)$ is the impurity position inside the ring and Δ is the two-dimensional Laplacean operator. The confining potential, $V(r)$, combines the parabolic and inverse square potential functions:

$$V(r) = \frac{\hbar^2}{2m^*}\frac{\lambda^2}{r^2} + \frac{1}{2}m^*\omega_0^2 r^2. \tag{3}$$

The dimensionless parameter, λ, characterizes the strength of the inverse square potential which describes the forbidden hollow region inside the ring [25, 26] and ω_0 is the confinement frequency of the parabolic potential. As a result, the inner radius $R_1 = \sqrt{\hbar^2\lambda^2/2m^*V_0}$ corresponds to the value of *r* where this term is equal to V_0 (the barrier potential for electrons in GaAs QR), whereas the outer radius $R_2 = \sqrt{2V_0/m^*\omega_0^2}$ is associated with the second term of the potential [25]. The application of an in-plane electric field and/or the presence of a Coulomb term breaks the axial symmetry and cannot be found analytical solutions for the eigenfunctions of the Hamiltonian (1). To obtain the eigenenergies E_j and the corresponding wavefunctions $\Psi_j(x, y)$ of the perturbed Hamiltonian we used a finite difference method and processed the data in MatLab environment. The convergence criterion was set to a relative tolerance of 10^{-6} and the results were get after 3 mesh refinements.

The change in the electron distribution within the nanostructure induced by a static electric field is described by the so called static dipole polarizability. It is well known that the quantum-confined Stark effect leads to a shift of the ground state energy given by:

$$\Delta E(F) = E(F) - E(F=0) = -\vec{p}\cdot\vec{F} - \frac{1}{2}\beta F^2. \tag{4}$$

This quadratic form follows from a second-order perturbation theory with $\vec{p}$ the dipole moment and β the polarizability of the system [27].

For F along the x-direction β is given by [27- 29]:

$$\beta = -\frac{e}{F}\left[\langle x\rangle_F - \langle x\rangle_{F=0}\right] = -\frac{e}{F}\left[\langle\Psi_1(\bar{r})|x|\Psi_1(\bar{r})\rangle_{F\neq 0} - \langle\Psi_1(\bar{r})|x|\Psi_1(\bar{r})\rangle_{F=0}\right] \quad (5)$$

and $p = \beta F$.

We complete our study by examining the optical properties of impurity in a quantum disc subjected to a lateral electric field. An important physical quantity which provides a qualitative understanding of the intraband transitions in quantum systems is the oscillator strength (OS), which may be regarded as an indicator of how strongly the system interacts with radiation field. The oscillator strength for a transition from level i to level n is obtained from standard formula as [30]

$$f_{ni} = \frac{2m_0}{\hbar^2}\Delta E_{in}\left|\mu_{in}\right|^2 \quad (6)$$

where $\Delta E_{in} = E_n - E_i$ is the energy difference between the electron states and

$$\mu_{ij} = \left\langle\Psi_i\,(x,y)\middle|r\middle|\Psi_j\,(x,y)\right\rangle \quad (7)$$

is the transition matrix element in the dipolar approximation. In Eq. (7), r refers to x for a x-polarized (along the electric field direction) and y for a y-polarized incident light, respectively.

3. Results and discussion

The system of our study corresponds to a flat two-dimensional GaAs region with a thickness L_z, surrounded by a $Ga_{0.6}Al_{0.4}As$ material. The adjustable parameters of the ring potential are chosen as $\lambda^2 = 4$ and $\omega_0 = 20$ THz, so that the carriers are confined between the inner radius R_1=2.7 nm and the outer radius R_2=62.8 nm, corresponding to $V_o \approx$ 300 meV. As in Ref. [25] we have assumed a vertical size L_z =2 nm, so that z-motion is "frozen" in its ground state and all features of the spectrum are essentially determined by the motion in the xy-plane. Three positions of the hydrogenic donor within the structure are considered in our study: $(r_0,0)$, $(-r_0,0)$, and $(0,r_0)$, where $r_0 = \sqrt{\hbar\lambda/m^*\omega_0}$ =13.19 nm is the position of the confining potential minimum at zero electric field (see Fig. 1).

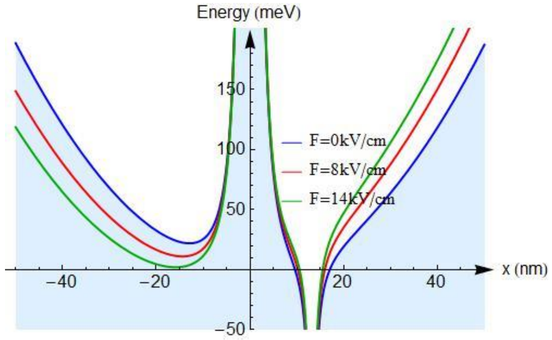

Fig. 1 The confinement potential energy along *x*-direction (including the Coulomb interaction) in a GaAs QR for different electric field strengths. Results are for $\lambda = 2$, $\omega_0 = 20$ THz and the impurity placed in $(r_0, 0)$.

The application of an external electric field to the QR leads to larger or smaller confinement of the impurity electron, depending on the impurity position. This is due to the interplay between the Coulomb attraction and the field-induced distortion of the potential. As Fig. 1 shows, the field induce a lowering of the parabolic potential barrier along the negative *x*-direction. Combined with the strong localization of the ground state wave function around the Coulomb center, the geometrical confinement variation determines the increase (decrease) of the E_1 with F when an impurity is placed at positive (negative) x_i. A similar variation of the E_1 with the change in the donor position along the *x*-axis was reported by Acosta *et al*. [25] for a QR under in-plane electric fields. Instead, for the impurity placed on *y*-axis the ground state energy variation is significantly smaller compared to the previously discussed case due to the smaller change in the confinement potential.

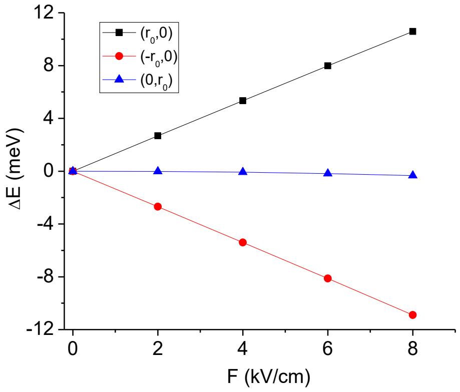

Fig. 2. Stark shift as a function of the electric field strength for off-center donors in a GaAs QR. The impurity is located at three different positions: $(r_0, 0)$, $(-r_0, 0)$, and $(0, r_0)$, respectively.

The Stark shift dependence on the donor location is plotted in Fig. 2 as a function of the electric field. For impurity placed along (opposite) field direction, the energy shift linearly increases (decreases) with the electric field. As expected, for a donor localized in $(0, r_0)$, when the charge distribution is less sensitive to the field influence, the Stark shift is not significantly changed by the electric field.

Fig. 3 displays the variation of the polarizability and the dipole moment for a shallow donor in a QR versus the electric field for different values of the impurity position.

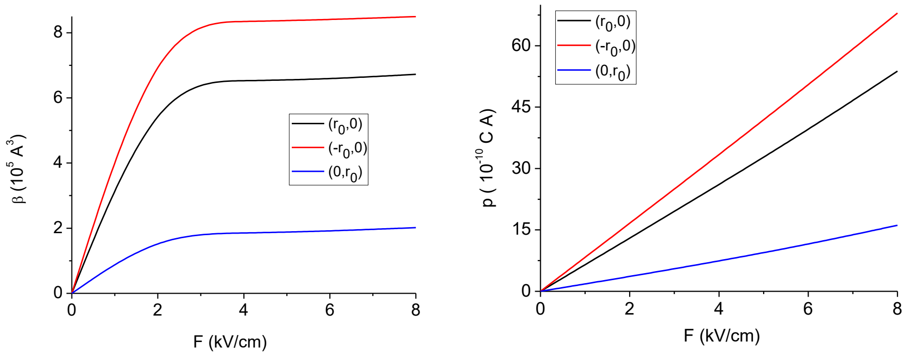

Fig. 3. The impurity polarizability (left) and dipole moment (right) as a function of the lateral electric-field amplitude *F*. Same impurity positions as in Fig. 2 are considered.

One observes that the polarizability increases as the applied field is increased and tends to saturation for rather small values of the electric field. The saturation for these three impurity positions has different explanations: (i) for localization along the field direction (black curve) it is the results of the competition between the Coulomb attraction (which keeps the electron cloud around the donor center, in the right side of the ring) and electric field effect (which pushes the carriers in the opposite (-*x*) direction. (ii) when the impurity is placed at $(-r_0, 0)$ (red curve), the behavior results from the strong confinement under combined effect of the Coulomb potential and the field-induced leakage of the electron wave function towards the left region, where the well bottom is shifted to lower energies. This increased confinement of the electron cloud leads to the poor impurity deformability under the electric field influence. (iii) For impurity placed along *y*-direction (blue curve) the polarizability is weaker because the effect of the field-related changes in the electron-impurity localization is small, as discussed before. All the curves in Fig. 3(b) follow the same trend: the dipole moments show a roughly linear increase with electric field, more distinct for the donor at $(-r_0, 0)$. As expected, the impurity placed along or perpendicular to field direction (black and blue curves in Fig. 3(b)) becomes less sensitive to field variations due to a weaker quantum confinement. Their ground states are more delocalized and consequently, the corresponding wave functions are spreaded over larger regions of the ring, exhibiting slighter field-induced changes.

The oscillator strength given by Eq. (6) depends on the transition energies $\hbar\omega_{ij}$ and and the square of the transition matrix element, $|\mu_{in}|^2$. Dependences of these quantities on the applied electric field is plotted in Figs. 4 and 5 for the same impurity positions as in Fig. 2.

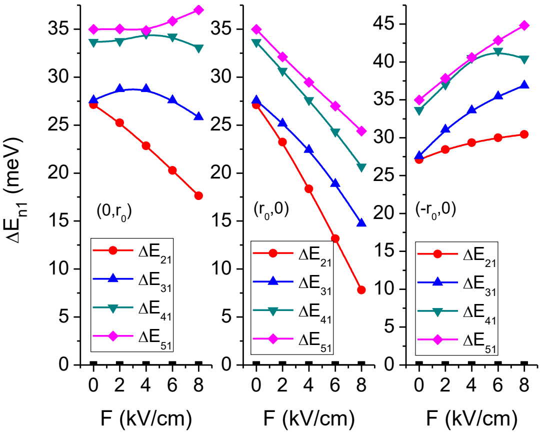

Fig. 4. Energy differences ΔE_{n1} versus applied electric field for the same impurity positions as in Fig. 2.

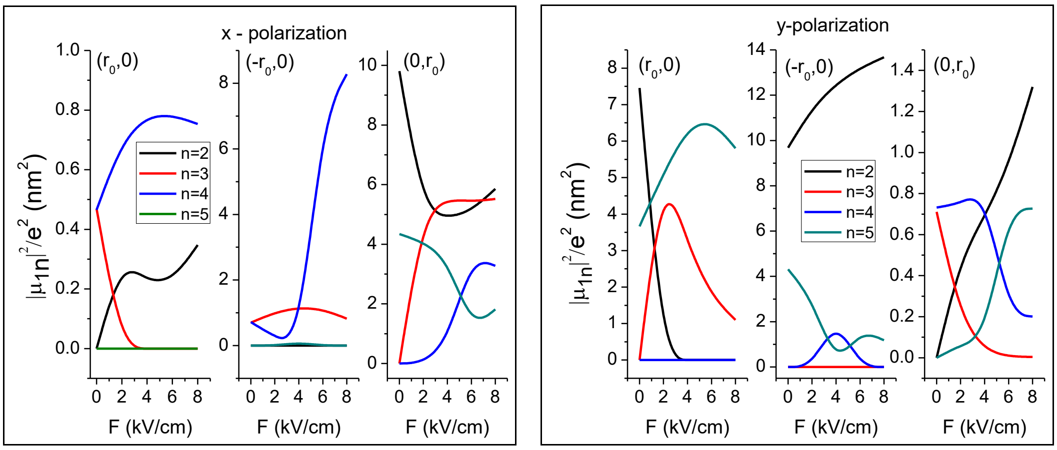

Fig. 5. The transition matrix element $|\mu_{1n}|^2$ versus the applied electric field. Two polarizations of the incident light and three impurity position in the QR are considered.

Dependence of the oscillator strength f_{n1} for the studied $1 \rightarrow n$ interlevel transition on the electric field is presented in Fig. 6. Three impurity positions in the QR and two polarizations of the incident light are considered.

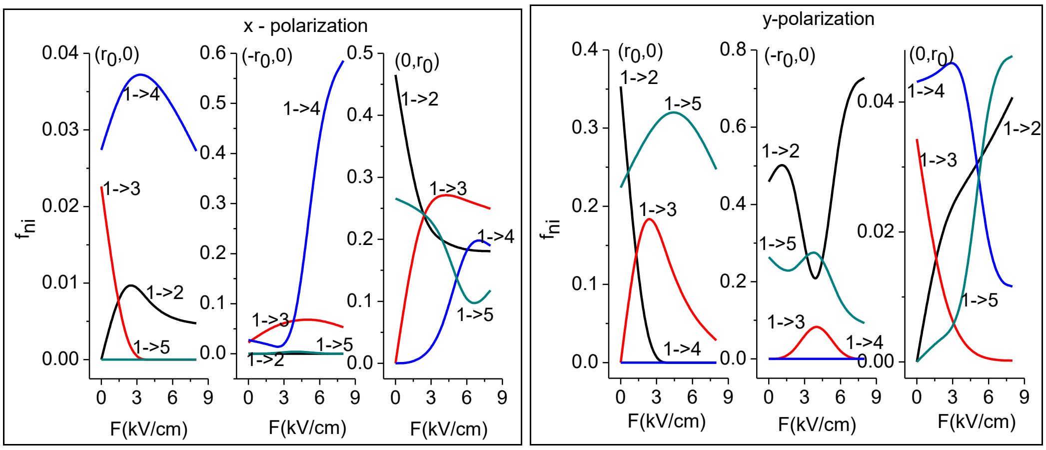

Fig. 6. Oscillator strength for the ground-to-lowest excited states transitions versus applied electric field for *x*- (left panel) and *y*- (right panel) polarization of the incident light. The same three impurity positions as in Fig. 2 are considered.

By comparing the results in two panels we notice that $|\mu_{1n}|^2$ (which is very sensitive to the incident light polarization) is the dominant term in the OS behavior. For example, when the impurity is placed in right side of the ring, the intensity of interlevel transitions induced by *y*-polarized light is at least one order of magnitude larger than those corresponding to a *x*-polarization. The situation changes for the $(0, r_0)$ location, when OS has appreciable values for the *x*-polarization. On the other hand, for a donor placed on left side of the ring the interlevels transitions show a very different behavior (see middle figures in panels). We note that the transitions $1 \rightarrow 2$ and $1 \rightarrow 5$, dominant for the *y*-polarized light, becomes practically forbidden for a *x*-polarization. Instead, the oscillator strength of 1->4 interlevel transition (which vanishes for a *y*-polarized light) dramatically increases with *F* for the *x*-polarized incident radiation. Such polarization peculiarities of nanorings optical characteristics can find the practical utilization for the modern optoelectronic devices.

6. Conclusions

In this paper, we have theoretical analyzed the effect of a lateral electric field on the Stark-shift, polarizability and oscillator strength associated to off-center donor impurities in a GaAs disc-shaped quantum ring. Using the finite difference method in the effective mass approximation, we found that in the range of the moderate field values the Stark-shift exhibits a linear dependence, and the impurity polarizability tends to saturation values. The behavior of the oscillator strength associated with impurity interlevel transitions shows a strong dependence both on the incident light polarization and on the impurity position within the structure. We found that for the impurity placed in right side of the ring the intensity of interlevel transitions induced by *y*-polarized light is at least one order of magnitude larger than those corresponding to a *x*-polarization. The situation changes for the $(0, r_0)$ location, when OS has appreciable values for the *x*-polarization. Because of the close relationship between the intraband absorption and the corresponding oscillator strength, our results should provide useful guidance for the design of more efficient quantum-ring infrared photodetectors.

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