SEVERAL ASPECTS ABOUT FRACTALITATY ROLE IN THE DYNAMICS OF COMPLEX SYSTEMS

Mihaela Viviana IVAN[1], Andrei ZALA [2], Andrei AGOP[3], Elena PUIU[4], Dorin VĂIDEANU[5], Ion PALAMACIUC[6], Dragoş TEODOR IANCU[7*], Radu CRIŞAN-DABIJA[8]

In the framework of the Scale Relativity Model with an arbitrary constant fractal dimension, several characteristic dynamics of complex systems induced only by the "fractal force" are presented. In this case, fractal velocity field is described both by topological solitons of kink type and by nontopological soliton varieties of breather type. The existence of such "nonlinearities" has as consequence either self-structuring effects by generating patterns or chaotic effects. Such behaviors are illustrated by analyzing the dynamics of blood, considered fractal fluid.

Keywords: nondifferentiability, complex system, fractal, solitons, kink solitons

1. Introduction

Complex systems are composed of many interacting structural units. The evolution of the complex systems cannot be predicted simply by analyzing the behaviour of individual elements or by superposing their individual evolutions [1, 2]. Even in the case of singular units the analysis can be very complicated, since the nonlinear dynamics of a system highly sensitive to the initial condition can evolve to chaos [3]. Therefore, the global evolution of a complex system is determined by the manner in which individual elements interact each other.

[1] Assoc. Prof., Cardiology Dept., "Victor Babeş" University of Medicine and Pharmacy Timișoara, Romania, e-mail: m.ivan@yahoo.com
[2] Assist., Electrical Engineering Dept., "Gheorghe Asachi" Technical University of Iasi, Romania, e-mail: andrei_zala@yahoo.com
[3] Assist., Material Science and Engineering Dept., "Gheorghe Asachi" Technical University of Iasi, Romania, e-mail: a.agop@yahoo.com
[4] Assist., Machine Manufacturing Technology Dept., "Gheorghe Asachi" Technical University of Iasi, Romania, e-mail: e.puiu@yahoo.com
[5] Assist., Physics Dept., "Al. I. Cuza" University Iași, Romania, e-mail: naturaone@gmail.com
[6] Assist., Material Science and Engineering Dept., "Gheorghe Asachi" Technical University of Iasi, Romania, e-mail: palamaciuc@yahoo.com
[7] *Prof., Medical Oncology and Radiotherapy Dept., "Grigore T. Popa" University of Medicine and Pharmacy , Iași, Romania, e-mail: dt_iancu@yahoo.com (corresponding author)
[8] Prof., Pneumology Dept., "Grigore T. Popa" University of Medicine and Pharmacy, Iași, Romania, e-mail: dabija@yahoo.com

In the classical concepts, the theoretical models (hydrodynamic, kinetic, etc. [4,5]) of complex systems are realized assuming that the dynamics of individual elements are characterized by continuous and differentiable motion variables (energy, momentum, density etc.), exclusively dependent on the spatial coordinates and time.

In reality, the complex system dynamics is much more complicated and the classical theoretical models failed in the attempt to explain all the concerned aspects, as illustrated by the experimental observations.

These difficulties can be overcome in a complementary approach, using fractal concepts, defined for the first time by Mandelbrot [6]. He introduced the term "fractal" to describe the"exotic" shapes that did not fit the patterns of Euclidean geometry, i.e. irregular geometrical objects, cells of living organisms, human arterial, neural network, convoluted surface of the brain etc., that possess invariance with respect to the scale transformations which can be captured well by the fractal geometry. In this context, the fractal analysis has proven to be a useful tool describing various systems from physics and chemistry [7-9], biology and medicine [10,11], econophysics [12,13].

Moreover, the analysis of complex systems evolution showed that most of them are nonlinear and, therefore, new mathematical tools were required. These have been provided by the Scale Relativity Theory (SRT) [14, 15] and by Extended Scale Relativity Theory (ESRT) [16], i.e. the Scale Relativity Theory with an arbitrary constant fractal dimension.

These theories consider that the motions of the complex systems structural units take place on continuous but non-differentiable curves (fractal curves). In this situation, the Euclidean dynamics of a complex system subjected to external constraints is replaced by a fractal dynamics characterizing the same system free of any external constraints. More precisely, the constrained motions in the Euclidian space, i.e. on continuous and differentiable curves, are substituted by free, independent motion (without constrains) in a fractal space, i.e. on continuous, but non-differentiable (fractal) curves (for details by means of applications, see [17-21])

Therefore, non-differentiability becomes a fundamental property of the complex system dynamics. In such conjecture, a correspondence between the trajectories can be established. Then, for specific scales that are large with respect to the inverse of the highest Lyapunov exponent [22], the deterministic trajectories are replaced by a collection of potential trajectories, while the concept of definite positions is substituted by that of the probability density. Moreover, the complex systems structural units may be reduced and identified with their own trajectories so that the complex system will behave as a special fluid lacking interactions (via their geodesics in a fractal space). We have called such fluid a "fractal fluid" (for details through its implications, see [23-27])

Let us note that there are currently a great number of works describing the behaviours of complex systems in different forms, from which most interesting, related to the application presented are drug delivery systems, in various formulations: microparticles [28], hydrogels [29-31], magnetic nanoparticles [32] All of these models employed the ESRT in order to develop mathematical models for in vitro drug release mechanisms.

Taking into account the above, the role of „fractal force” in the dynamics of complex systems is analyzed, using the ESRT model.

2. Hallmarks of non-differentiability

In such a framework, some consequences of non-differentiability both in the usual space (of the space and time coordinates) and in the scale space are evident [14-16]:

i) any continuous, but non-differentiable curve of the complex system structural units (fractal curve) is explicitly dependent of scale resolution δt, i.e. its length tends to infinity when δt tends to zero;

ii) the physics of the complex system phenomena is related to the behaviour of a functions set during the zoom operation of the scale δt. Then, through the substitution principle, δt will be identified with dt, i.e, $\delta t = dt$. Consequently, it will be considered as an independent variable;

iii) the complex system dynamics is described through fractal variables. As consequence, the velocity field, both in the usual space and in scale space, becomes a complex variable dynamics, with the form:

$$\hat{V}^l = V_D^l - iV_F^l, \qquad i = \sqrt{-1} \tag{1}$$

where the real part, V_D^l, is the differentiable velocity and the imaginary one, V_F^l, is the non-differentiable (fractal) velocity;

iv) the differential of the spatial coordinate field, $d_\pm Y^i$, is expressed as the sum of two differentials, one of them being differential part $d_\pm y^i$ and the other one being scale fractal part, $d_\pm \sigma^i$, i.e.:

$$d_\pm Y^i = d_\pm y^i + d_\pm \sigma^i \tag{2}$$

The sign ”+” corresponds to the forward process, while the sign ”-“ to the backwards one;

v) the fractal part of the spatial coordinate field satisfies the fractal equation:

$$d_\pm \sigma^i(t,dt) = \lambda_\pm^i (dt)^{1/D_F} \tag{3}$$

where D_F defines the fractal dimension of the fractal motion curve and $\lambda_\pm^i$ are constant coefficients that indicates the fractalisation type;

vi) an infinite number of fractal curves can be found relating any pair of points, both in the usual space and in scale space. Then, any external constraint is interpreted as a selection of fractal curves, corresponding to the maximum of the probability density;

vii) the complex system dynamics, both in the usual space and in scale space, can be described through a scale covariant derivative:

$$\frac{\hat{d}}{dt} = \partial_t + \hat{V}^l \partial_l - \frac{1}{4}(dt)^{(2/D_F)-1} D^{lk}\partial_l\partial_k \tag{4}$$

where:

$$D^{lk} = \left(\lambda_+^l\lambda_+^k - \lambda_-^l\lambda_-^k\right) - i(\lambda_+^l\lambda_+^k + \lambda_-^l\lambda_-^k)$$
$$\partial_l = \frac{\partial}{\partial Y^l};\partial_l\partial_k = \frac{\partial^2}{\partial Y^l \partial Y^k} \tag{5}$$

In the previous relations the indices l,k take the values $1,2,3$ in the usual space, while in the scale space they have an arbitrary dimension imposed by the intrinsic structure of the complex system.

Considering now the functionality of a generalized covariance principle [14-16], the standard derivative operator d/dt is replaced by the non-differentiable operator $\hat{d}/dt$.

Under these conditions, applying the operator (4) to the complex velocity field (1), in the absence of any external constraint and for motions on Levy curves [5], which implies the restriction

$$\lambda_+^i\lambda_+^l = -\lambda_-^i\lambda_-^l = 2\lambda\delta^{il} \tag{6}$$

with λ the fractal-nonfractal transition coefficient (for details see [9, 14-16]) and δ^{il} the Kronecker tensor, the fractal equation of the motion (geodesics equation) have the following form:

$$\frac{\hat{d}\hat{V}^i}{dt} = \partial_t \hat{V}^i + \hat{V}^l \partial_l \hat{V}^i - i\lambda(dt)^{(2/D_F)-1}\partial^l\partial_l \hat{V}^i = 0 \tag{7}$$

Previous result shows that, both in the usual space and in scale space, the local “acceleration”, the ”convection” and the “dissipation” make their balance in any point of non-differentiable curve. Moreover, the presence of the complex coefficient of viscosity type indicates that the complex system is a rheological medium, so it has memory, as a datum, by its own structure.

In equation (7), by separating the motions on differential and fractal scale resolutions, it results:

$$\frac{\hat{d} V_D^i}{dt} = \partial_t V_D^i + V_D^l \partial_l V_D^i - \left[V_F^l - \lambda (dt)^{(2/D_F)-1} \partial^l\right] \partial_l V_F^i = 0$$
$$\frac{\hat{d} V_F^i}{dt} = \partial_t V_F^i + V_D^l \partial_l V_F^i - \left[V_F^l - \lambda (dt)^{(2/D_F)-1} \partial^l\right] \partial_l V_D^i = 0 \tag{8}$$

3. Dynamics only at fractal scale resolutions

Further, let's analyze which type of dynamics "hide" the cancellation of "fractal force".

$$V_F^l \partial_l V_F^i = \lambda (dt)^{(2/D_F)-1} \partial^l \partial_l V_F^i \tag{9}$$

under the condition that the fractal fluid at non-differentiable scale is incompresible:

$$\partial_i V_F^i = 0 \tag{10}$$

Finding the solutions for these equations can be relatively difficult, due to the fact that the system equations are non-linear. However, there is an analytical solution of this system, in the particular case of a stationary flow in a plane symmetry (x, y). In these circumstances, equations (9) and (10), with $\boldsymbol{V}_F = (V_x, V_y, 0)$ take the form:

$$V_x \frac{\partial V_x}{\partial x} + V_y \frac{\partial V_x}{\partial x} = \lambda (dt)^{(2/D_F)-1} \frac{\partial^2 V_x}{\partial y^2} \tag{11}$$

$$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} = 0 \tag{12}$$

where $V_x = V_x(x, y)$ is the fractal velocity along axis Ox, $V_y = V_y(x, y)$ is the fractal velocity along Oy axis. The boundary conditions of the flow are:

$$\lim_{y \to 0} V_y(x, y) = 0, \qquad \lim_{y \to 0} \frac{\partial V_x}{\partial y} = 0, \quad \lim_{y \to \infty} V_x(x, y) = 0 \tag{13}$$

and the flux momentum per length unit is constant:

$$\Theta = \rho \int_{-\infty}^{+\infty} V_x^2 dy = const. \tag{14}$$

Using the method from [33] for resolving the equations (11) and (12), with the limit conditions (13) and (14), the following solutions result:

$$V_x = \frac{\left[1.5\left(\frac{\Theta}{6\rho}\right)^{2/3}\right]}{\left[\lambda (dt)^{(2/D_F)-1} x\right]^{1/3}} \cdot \sec h^2 \frac{\left[(0.5y)\left(\frac{\Theta}{6\rho}\right)^{1/3}\right]}{\left[\lambda (dt)^{(2/D_F)-1} x\right]^{2/3}} \tag{15}$$

$$V_y = \frac{\left[4.5\left(\frac{\Theta}{6\rho}\right)^{2/3}\right]}{\left[3\lambda(dt)^{(2/D_F)-1}x\right]^{1/3}} \cdot \left[\frac{\left[y\left(\frac{\Theta}{6\rho}\right)^{1/3}\right]}{\left[\lambda(dt)^{(2/D_F)-1}x\right]^{2/3}} \cdot \sec h^2 \frac{\left[(0.5y)\left(\frac{\Theta}{6\rho}\right)^{1/3}\right]}{\left[\lambda(dt)^{(2/D_F)-1}x\right]^{2/3}} - \tanh \frac{\left[(0.5y)\left(\frac{\Theta}{6\rho}\right)^{1/3}\right]}{\left[\lambda(dt)^{(2/D_F)-1}x\right]^{2/3}}\right] \tag{16}$$

Relations (15) and (16) suggest that the fractal velocity field of the fractal fluid is highly non-linear by means of soliton-breather and soliton-kink type solutions (for details on such solutions, see [22, 34]). Given the structural complexity of the fluid, an accurate way of writing relations (15) and (16) will be the one in which we assign indexes for each component.

For $y = 0$, we obtain, from relation (15), the flow critical velocity in the form:

$$V_x(x, y=0) = V_c = \frac{\left[1.5\left(\frac{\Theta}{6\rho}\right)^{2/3}\right]}{\left[\lambda(dt)^{(2/D_F)-1}x\right]^{1/3}} \tag{17}$$

while relation (14), taking into account (17), becomes:

$$\Theta = \rho\int_{-\infty}^{+\infty} V_x^2(x, y)dy = \int_{-d_c}^{+d_c} V_c^2(x, 0)dy \tag{18}$$

so that the critical cross section of the strains lines tube is given by:

$$d_c(x, y=0) = \frac{\Theta}{2\rho V_c^2} = 2.42\left[\left[\lambda(dt)^{(2/D_F)-1}x\right]^{2/3}\right]\left(\frac{\rho}{\Theta}\right)^{1/3} \tag{19}$$

Relations (15) and (16) can be strongly simplified if we introduce the normalized quantities:

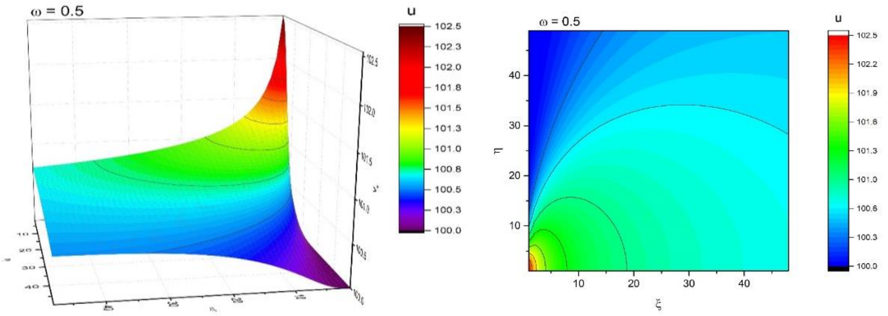
$$\xi = \frac{x}{x_0}, \eta = \frac{y}{y_0}, u = \frac{V_x}{w_0}, v = \frac{V_y}{w_0}, \Omega = \frac{\left(\frac{\Theta}{6\rho}\right)^{2/3}}{w_0\left[\lambda(dt)^{(2/D_F)-1}x_0\right]^{1/3}},$$
$$\omega = \frac{\left(\frac{\Theta}{6\rho}\right)^{1/3} y_0}{\left[\lambda(dt)^{(2/D_F)-1}x_0\right]^{2/3}} \tag{20}$$

where x_0, y_0, w_0 are specific lengths and, respectively, the specific velocity of the flow of the fractal fluid. It results that:

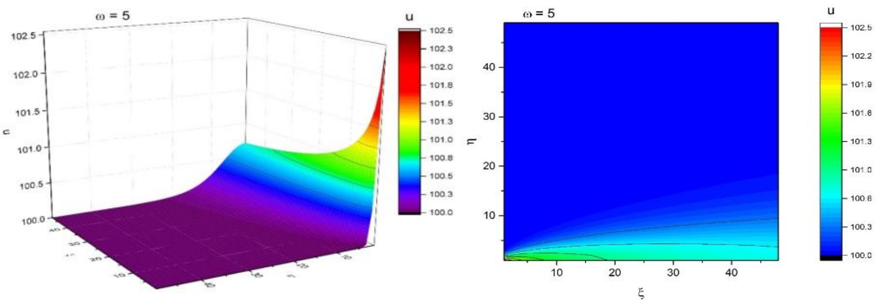
$$u(\xi,\eta)=\frac{1.5\Omega}{\xi^{1/3}}\operatorname{sec}h^2\left(\frac{0.5\Omega\omega\eta}{\xi^{2/3}}\right) \tag{21}$$

$$v(\xi,\eta)=\frac{4.5^{2/3}}{3^{1/3}}\frac{\Omega}{\xi^{1/3}}\left[\frac{\omega\eta}{\xi^{2/3}}\operatorname{sec}h^2\left(\frac{0.5\Omega\omega\eta}{\xi^{2/3}}\right)-\tanh\left(\frac{0.5\Omega\omega\eta}{\xi^{2/3}}\right)\right] \tag{22}$$

We present in Fig. 1a,b – 2a,b the dependence of the normalized velocity field u on the normalized spatial coordinates ξ,η for different nonlinearity degrees ($\omega = 0.5$ in Fig. 1 and $\omega = 5$ in Fig. 2). The results showcase that the velocity field on the flow direction (ξ) is affected in a weak manner by the nonlinearity degree (the velocity always decreases on the flow axes regardless of the nonlinearity degree). On the other hand, the flow direction (η) is strongly affected. The flow starts from constant values on the η axis, and with the increase of ω, preferential flow direction can be identified.

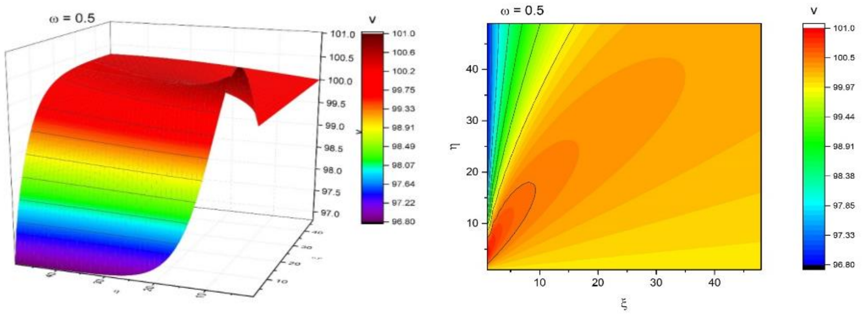


Figs. 1a, b: The dependence of the normalized velocity field *u* on the normalized spatial coordinates *ξ, η* in three dimensional (a) and contour plot (b) representations, for the nonlinearity degree *ω* = 0.5

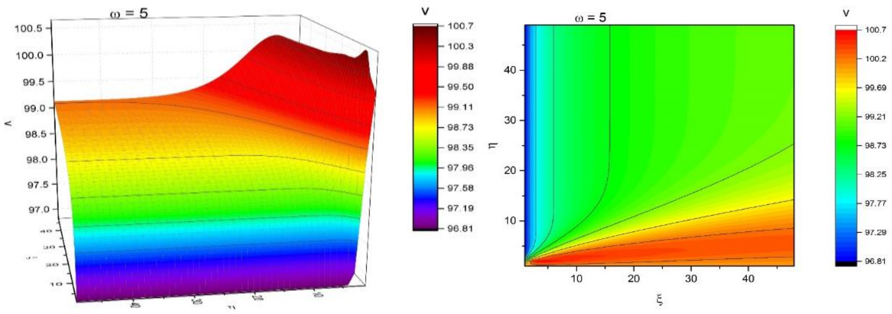


Figs. 2a, b: The dependence of the normalized velocity field *u* on the normalized spatial coordinates *ξ, η* in three dimensional (a) and contour plot (b) representations, for the nonlinearity degree *ω* = 5

In Fig. 3a,b – 4a,b the dependences of the normalized velocity field v on the normalized spatial coordinates ξ, η for different nonlinearity degrees ($\omega = 0.5$ in Fig. 3 and $\omega = 5$ in Fig. 4) are represented. For small nonlinearity degrees the variations of the velocity field have similar behaviors on both directions (ξ, η), while for higher values of the nonlinearity degree these variations are only focused on a single direction (ξ).



Figs. 3a, b: The dependence of the normalized velocity field *v* on the normalized spatial coordinates ξ, η in three dimensional (a) and contour plot (b) representations, for the nonlinearity degree ω = 0.5



Figs. 4a, b: The dependence of the normalized velocity field *v* on the normalized spatial coordinates ξ, η in three dimensional (a) and contour plot (b) representations, for the nonlinearity degree ω = 5

4. Blood flow dynamics through fractal fluid approach

Blood (composed of blood cells suspended in blood plasma) is a body fluid that transport both necessary substances such as nutrients and oxygen to the cells and metabolic waste products away from the same cells. Plasma, which constitutes 55% of a blood’s fluid, is mostly water, containing dissipated proteins, glucose, mineral ions, hormones, carbon dioxide, other molecules (like cholesterol) and blood cells themselves. Blood circulation around the body through blood vessels is assured by the pumping action of the heart.

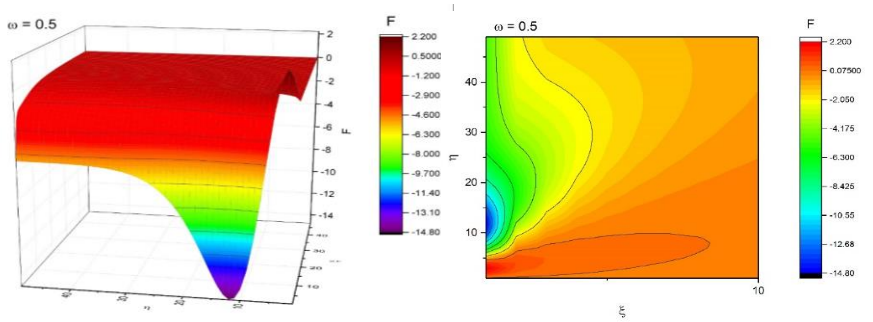
From an anatomical and histological point of view, blood is considered a specialized form of connective tissue, given its origin in the bones and the presence of potential molecular fibers in the form of fibrinogen.

Given that the circulatory system has a fractal structure, it is expected its functionality to be also fractal. This allows us to assimilate the dynamics of the blood flow with the one of a fractal fluid. In such context, although the velocity fields will remain the same as the one presented in Fig. 1-4, the force that the fluid will exercise to the walls of the flow vessels is of great importance for the understanding of arterial occlusion and other circulatory system diseases.

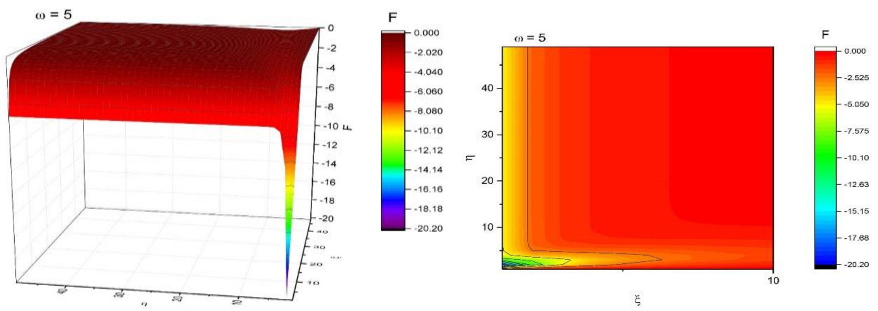
In our case the normalized force is given by the relation:

$$F \approx \partial_\eta u - \partial_\xi \upsilon \qquad (23)$$

In Fig. 5a,b-6a,b are represented the normalized force field evolutions on the two-flow direction (ξ,η) for different nonlinearity degrees ($\omega = 0.5$ in Fig. 5 and $\omega = 5$ in Fig. 6). It results that with the increase of the nonlinearity of the fluid the force towards the walls increases. This can be a starting point for understanding the complexity of the mechanisms involved in the arterial occlusion.



Figs. 5a,b: The dependence of the normalized velocity field *F* of a blood flow, on the normalized spatial coordinates *ξ*, *η* in three dimensional (a) and contour plot (b) representations, for the nonlinearity degree *ω* = 0.5



Figs. 6a,b: The dependence of the normalized velocity field *F* of a blood flow, on the normalized spatial coordinates *ξ*, *η* in three dimensional (a) and contour plot (b) representations, for the nonlinearity degree *ω* = 5

The proposed theory has the advantage that it can explain the atherogenesis process, from a fractal point of view, basically “molding” to the classical anatomical and histopathological descriptions.

6. Conclusions

The present paper proposes a fractal model for the dynamics analysis of complex fluids flows. The fractal hydrodynamic equations were obtained and applied for the laminar flow of a fractal fluid.

An application for the blood flow was proposed. The results revealed the directional flow towards the walls, which can explain the thickening effect which is one of the sources of arteriosclerosis.

We can thus say that fractality represents the mathematical and semantic quintessence for complex biological system evolution, both by itself and in interaction with other complex systems, a process that can be characterized perfectly by fractal physics, thus physics becoming more of a component rather than an explanation for the complex biological system.

R E F E R E N C E S

[1]. *M. Mitchell*, Complexity: A Guided Tour, Oxford University Press, Oxford, 2009

[2]. *R. Badii and A. Politi*, Complexity: Hierarchical Structure and Scaling in Physics, Cambridge University Press, 1997

[3]. *B. M Mihalcea*, Nonlinear harmonic boson oscillator, Physica Scripta, vol. **T140**, 2010 014056

[4]. *Y. H. Thomas*, Multi-scale Phenomena in Complex Fluids: Modelling, Analysis and Numerical Simulations, World Scientific Publishing Company, Singapore, 2009

[5]. *O.D. Michel and B. G. Thomas*, Mathematical Modelling for Complex Fluids and Flows, Springer, New York, 2012

[6]. *B. B. Mandelbrot*, The Fractal Geometry of Nature (Updated and augm. ed.), W. H. Freeman, New York, 1983

[7]. *J. Federer and A. Aharoner*, Fractals in physics, North Holland, Amsterdam, 1990

[8]. *B M Mihalcea, G Vişan*, Nonlinear ion trap stability analysis, Physica Scripta, vol. **T140**, 2010, 014057

[9]. *M. S. El Naschie, O. E.Rossler, and I. Prigogine*, Quantum Mechanics, Diffusion and Chaotic Fractals, Elsevier, Oxford, 1995

[10]. *A. T. Winfree*, The Geometry of Biological Time, Springer 2nd edition, New York, 2000

[11]. *C. Stan, C. Astefanoaei, E. Pretegiani, et al*., Nonlinear analysis of saccade speed fluctuations during combined action and perception tasks, Journal of Neuroscience Methods, **vol. 232,** 2014, pp. 102-109

[12]. *E I Scarlat, C. Stan, and C. P. Cristescu*, Self-similar characteristics of the currency exchange rate in an economy in transition, Physica A: Statistical Mechanics and its Applications, **vol. 379**, no. 1, 2007, pp. 188-198

[13]. *C P Cristescu., C.Stan, E I. Scarlat*, Multifractal analysis of the dynamics of the Romanian exchange rate ROL-USD during the transition period, UPB Scientific Bulletin, Series A, **vol. 69**, no. 3, 2007, pp. 37-44

[14]. *L. Nottale*, Fractal Space-Time and Microphysics: Towards a Theory of Scale Relativity, World Scientific, Singapore, 1993
[15]. *L. Nottale*, Scale Relativity and Fractal Space-Time. A New Approach to Unifying Relativity and Quantum Mechanics, Imperial College Press, London, 2011
[16]. *I. Merches and M. Agop*, Differentiability and Fractality in Dynamics of Physical Systems, World Scientific, Singapore, 2016
[17]. *A. Timofte, I. Casian Botez, D. Scurtu, et al.*, System Dynamics Control through the Fractal Potential, Acta Physica Polonica A, **vol. 119**, no. 3, 2011, pp. 304-311
[18]. *O. Niculescu, P. Nica, S. Gurlui, et al.*, Experimental Investigations of Polymer Plasma Laser Ablation, Materiale Plastice, **vol. 46**, no. 3, 2009, pp. 336-338
[19]. *C. Popa, P. Vizureanu, I. Casian Botez,et al.*, Theoretical and Experimental Results on the Charge Transport in Plasma Structures Through Spontaneously Symmetry Breaking.New Transport Mechanisms in Composite Materials, Materiale Plastice, **vol. 46**, no. 2, 2009, pp. 144-148
[20]. *R. Ciobanu, R. Damian, I. Casian Botez*, Electromagnetic characterization of chiral auxetic metamaterials for EMC applications, Computer Standards&Interfaces, **vol. 32**, no. 3, 2010, pp. 101-109
[21]. *M. Agop, I. Casian-Botez, V. S. Birlescu, et al.*, On the Memorizing Ability of Nanostructures, Journal of Computational and Theoretical Nanoscience, **vol. 12**, no. 4, 2015, pp.682-688
[22]. *C. P. Cristescu*, Dinamică neliniară și haos. Fundamente teoretice și aplicații (Non-linear dynamics and chaos. Theoretical fundaments and applications), Romanian Academy Publishing House, Bucharest, 2008
[23]. *M. A. Paun, R. Hanu Cimpoesu, N. Cimpoesu, et al.*, Internal Friction Phenomena at Polymeric and Metallic Shape Memory Materials. Experimental and Theoretical Results, Materiale Plastice, **vol. 47**, no. 2, 2010, pp. 209-214
[24]. *M. Agop, P. Nica, P.D. Ioannou, et al.*, El Naschie's epsilon((infinity)) space-time, hydrodynamic model of scale relativity theory and some applications, Chaos Solitons & Fractals, **vol. 34**, no. 5, 2007, pp. 1704-1723
[25]. *M. Agop, C. Murgulet*, El Naschie's epsilon((infinity)) space-time and scale relativity theory in the topological dimension D=4, Chaos Solitons & Fractals, **vol. 32**, no. 3, 2007, pp.1231-1240
[26]. *M. Agop, P. D. Ioannou, P. Nica, et al.*, Fractal characteristics of the solidification process, Materials Transactions, **vol. 45**, no. 3, 2004, pp.972-975
[27]. *C. Nejneru, A. Nicuta, B. Constantin, et al.*, Dynamics Control of the Complex Systems via Nondifferentiability, Journal of Applied Mathematics, article number: 137056, 2013
[28]. *D. Magop, S. Băcăiţă, C. Peptu, M. Popa, M. Agop*, Non-differentiability at mesoscopic scale in drug release processes from polymer microparticles, Materiale Plastice, **vol. 49**, no. 2, 2012, pp. 101-105
[29]. *E. S. Bacaita, M. Agop***,** A multiscale mechanism of drug release from polymeric matrices: confirmation through a nonlinear theoretical model**,** Physical Chemistry Chemical Physics, **vol. 18**, 2016, pp. 21809-21816
[30]. *E. S. Bacaita, B. C. Ciobanu, M. Popa, et. al.*, Phases in the temporal multiscale evolution of the drug release mechanism in IPN-type chitosan based hydrogels, Physical Chemistry Chemical Physics, **vol. 16**, 2014, pp. 25896-25905
[31]. *V. Radu, S. Băcăiţă, A. Uliniuc, M. Popa, S. Susanu*, Fractal hydrodynamic model for drug release processes from starch based hydrogels, Materiale Plastice, **vol. 50**, no. 1, pp. 18-22, 2013.

[32]. *A. Durdureanu-Angheluta, S. Băcăiţă, V. Radu, et. al.,* Mathematical modelling of the release profile of anthraquinone-derived drugs encapsulated on magnetite nanoparticles, Revue Roumaine de Chimie, **vol. 58**, no. 2-3, 2013, pp. 217-22
[33]. *H. Schlighting*, Boundary layer theory, McGraw-Hill, New York, 1970
[34]. *E. A. Jackson*, Perspective of non-linear dynamics, Cambridge University Press, Cambridge, 1991