THE UNCERTAINTY ANALYSIS OF THE PIPELINE SYSTEM

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Pe parcursul studierii modelelor matematice a unor sisteme tehnice reale ne putem întâlni cu o incertitudine de anumit tip şi mărime. În cazul sistemelor de transport al lichidelor sursele incertitudinii parametrice pot fi diferenţele dintre parametrii sistemului tehnic, valorile caracteristice al regimului de funcţionare, respectiv compoziţia, parametrii fizici ai lichidului transportat. Lucrarea prezintă metoda de analiză a sensibilităţii parametrice a sistemelor de transport de lichide şi evaluarea rezultatelor de analiză obţinute prin exemplul unui sistem simplu. Aceste concluzii, experienţe pot fi folosite la analiza incertitudinii parametrice a sistemelor de conducte geotermale, cum ar fi incertitudinea caracteristicilor lichidelor

During mathematical model investigation of real technical systems we can meet any type and rate model uncertainty. In case of pipeline systems the sources of parameter uncertainties can be anomalies of technical system data, the mode of functioning values, composition and physical parameters of the fluid. The paper shows the methodology for sensitivity analysis and the discussion of its results by an easy pipeline system model case. These conclusions and experiences can be used to investigate parametrical uncertainties of geothermal pipeline systems, such as fluid characteristic's indeterminations.

Keywords: uncertainty analysis, mathematical model, pipeline system

List of Symbols

y	—	general dependent parameter;
x	—	general independent parameter;
K	—	general coefficient;
μ	—	dynamic viscosity $\left[\frac{\text{Ns}}{\text{m}^2}\right]$;
ρ	—	fluid density $\left[\frac{\text{kg}}{\text{m}^3}\right]$;
ν	—	kinematic viscosity $\left[\frac{\text{m}^2}{\text{s}}\right]$;

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c	—	average flow velocity $\left[\frac{m}{s}\right]$;
$\dot{V}$	—	volume flow rate $\left[\frac{m^3}{s}\right]$;
d	—	intern diameter $[m]$;
l	—	tube length $[m]$;
Re	—	Reynolds-number $[-]$;
λ	—	pipe loss coefficient $[-]$;
h'_{cs}	—	head loss of pipe $[m]$.
Δp_{cs}	—	the pressure loss of the pipe $[Pa]$;
h'_{sz}	—	the end loss of the pipe fitting $[m]$;
Δp_{sz}	—	the pressure loss of the pipe fitting $[Pa]$;
ξ	—	pipe fitting loss coefficient $[-]$;
$\underline{\underline{A}}$	—	the coefficient matrix of dependent variables;
$\underline{\underline{B}}$	—	the coefficient matrix of independent variables;
$\underline{\underline{D}}$	—	sensitivity coefficient matrix.

1. Introduction

During mathematical modeling of the real technical systems we can meet any type and rate model uncertainty [9]. They appear due to approximations of models or data inaccuracy. Classification of uncertainties, with respect to their sources, distinguishes between aleatory and epistemic ones. The aleatory uncertainty is an inherent data variation associated with the investigated system or the environment. Therefore it is named parametric uncertainty. Epistemic uncertainty is due to the lack of the knowledge of quantities, processes of the system or the environment. Aleatory uncertainty is primarily associated with objectivity but epistemic uncertainty may be comprised of substantial amounts of both objectivity and subjectivity [8].

In case of geothermal pipeline (for example heating) systems, parametrical model (system) uncertainties mean the indetermination of physical parameters of the fluid. These characteristics influence the system parameters such as loss at the ends, therefore required pump power.

Following Ferson and Tucker [2] the uncertainty analysis is a systematic study in which the neighborhood of alternative assumptions is selected and the corresponding interval of inferences is identified. According to Macdonald and Strachan [5], the sensitivity analysis is an important technique to determine the effect that uncertainties or model variations have on the model predictions. The analysis can be carried out from a simple level to a comprehensive treatment. In practice, sensitivity analysis is used in an ad hoc way in a lot of practical modeling studies.

Mahdavi studied various sources of the uncertainty in building performance simulation [6]. In his paper, the potential errors due to i) inaccurate building descriptions, ii) uncertain micro-climatic assumptions and iii) deficient building users' information are discussed, using original data and analysis.

Two mathematical models both dynamic and stationary, which are useful in the studying hydraulic systems are presented in a paper of Prodan and Iacob, [11]. Bucur and Isbăşoiu studied the influence of air pockets trapped in pipeline systems over the entire system pressure [1].

Following Gută et al. the simulating or the operating regime of systems or subsystems represents an important step in design process. Obtaining mathematical model helps in the decision making regarding the way of optimizing system [3]. Mirel et al. stated that, the geothermal waters are very valuable thermo-energetic resources [7]. The corrosive and hardness characteristics of geothermal waters — which are sources of geothermal water pipeline system uncertainties — can be eliminated by applying some specific treatment technologies according to water temperature, the chemical characteristics and the user's requirements.

The aim of this paper is to show the methodology of the sensitivity analysis and its possibility of use by an easy pipeline system model and discussions of results. These - basically theoretical — conclusions and experiences can be used to investigate parametrical uncertainties of the geothermal pipeline system, such as fluid characteristic's indeterminations.

The outline of the paper is as follows: Section 2 shows the sensitivity analysis. Section 3 presents an easy case study by a pipeline system model. Section 4 interprets the result of the sensitivity analysis. Section 5 summaries the paper and outlines the prospective scientific work of the author.

2. The Sensitivity Analysis

The essence of the sensitivity analysis is that the anomalies and variations of dependent system parameters are simulated by changing its independent (input and inner) variables. On the basis of the mathematical model of the investigated system one can determine how sensitive dependent system variables are to simulated changes. If only one independent variable is changed, the investigation will be called one-parameter sensitivity analysis. If the number of the changed independent variables is more than one, the several-parameter sensitivity analysis is used.

It is important to mention that changes of independent variables cannot be more than about 1 or 5 %, depending on the intensity of the original model nonlinearity. Depending on the nonlinearity of the original model, results of the sensitivity analysis can have differences from real influences of simulated changes. But these results show the direction and order of the magnitude of real simulated changes.

To determine the sensitivity coefficient as a first step, the total differential of both sides of the initial equation

$$y = f(x_1, x_2, \ldots, x_n) \tag{1}$$

should be formed:

$$dy = \frac{\partial f(x_1; x_2; \ldots x_n)}{\partial x_1} dx_1 + \ldots + \frac{\partial f(x_1; x_2; \ldots x_n)}{\partial x_n} dx_n \tag{2}$$

Then both sides of the last equation should be multiplied by same sides of the general equation and all elements should be multiplied by $\frac{x_i}{x_i}$:

$$\frac{dy}{y} = \frac{\partial f(x_1; x_2; \ldots x_n)}{\partial x_1} \frac{x_1}{f(x_1; x_2; \ldots x_n) x_1} dx_1 + \ldots + \frac{\partial f(x_1; x_2; \ldots x_n)}{\partial x_n} \frac{x_n}{f(x_1; x_2; \ldots x_n) x_n} dx_n \tag{3}$$

Introducing the sensitivity coefficients:

$$K_{y;x_i} = \frac{\partial f(x_1; x_2; \ldots x_n)}{\partial x_i} \frac{x_i}{f(x_1; x_2; \ldots x_n)} = \frac{\partial y}{\partial x_i} \frac{x_i}{y} \tag{4}$$

and considering

$$\frac{d\eta}{\eta} \approx \frac{\Delta \eta}{\eta} = \delta \eta$$

the following linear system can be achieved:

$$\delta y = K_{y;x_1} \delta x_{y;x_2} + \ldots + K_{y;x_n} \delta x_n \quad , \tag{5}$$

The equation mentioned above, shows how sensitive dependent system output parameters will be to uncertainties of input ones. For example, these uncertainties can occurr due to measurement inaccuracies.

If the investigated system has several dependent variables, the equations determined above can be written in the following matrix form:

$$\underline{\underline{A}} \delta \underline{y} = \underline{\underline{B}} \delta \underline{x} \quad , \tag{6}$$

where $\underline{\underline{A}}$ and $\underline{\underline{B}}$ are coefficient matrices of external and internal parameters of the investigated system.

Using the sensitivity coefficient matrix of investigated system,

$$\underline{\underline{D}} = \underline{\underline{A}}^{-1} \underline{\underline{B}} \tag{7}$$

the equation

$$\delta \underline{y} = \underline{\underline{D}} \delta \underline{x} \tag{8}$$

can be used for sensitivity investigations.

3. Creating Sensitivity Model (Case study)

In this case study, the pressure loss and end loss of two main pipeline system structural elements (linear pipe and pipe fitting) will be investigated. Therefore, the illustrative system consists of only one linear pipe and only one pipe fitting. The system was modeled in case of different Reynolds-number intervals (that is streams).

The kinematic viscosity of the fluid is:

$$\nu = \frac{\mu}{\rho} \quad , \tag{9}$$

the average flow velocity:

$$c = \frac{4\dot{V}}{d^2 \pi} \quad . \tag{10}$$

and the Reynolds number:

$$\mathrm{Re} = \frac{cd}{\nu} \quad . \tag{11}$$

The pipe loss coefficient can be determined, depending on Reynolds-number, by empirical equations in case of different Reynolds-number intervals [10].

If $Re < 2320$ then

$$\lambda_a = \frac{64}{\mathrm{Re}} \quad ; \tag{12a}$$

if $2320 < \mathrm{Re} < 8 \cdot 10^4$ then

$$\lambda_b = \frac{0{,}316}{\sqrt[4]{\mathrm{Re}}} \quad ; \tag{12b}$$

if $2 \cdot 10^4 < \mathrm{Re} < 2 \cdot 10^6$ then

$$\lambda_c = 0{,}0054 + 0{,}396\,\mathrm{Re}^{-0{,}3} \quad ; \tag{12c}$$

if $10^5 < \mathrm{Re} < 10^8$ then

$$\lambda_d = 0{,}0032 + 0{,}211\,\mathrm{Re}^{-0{,}337} \quad . \tag{12d}$$

The end loss of the pipe is

$$h'_{cs} = \frac{c^2}{2g} \frac{l}{d} \lambda \quad , \tag{13}$$

and its pressure loss

$$\Delta p_{cs} = \frac{\rho}{2} c^2 \frac{l}{d} \lambda \quad . \tag{14}$$

The end loss of the pipe fitting is

$$h'_{sz} = \frac{c^2}{2g} \xi \quad , \tag{15}$$

and its pressure loss

$$\Delta p_{sz} = \frac{\rho}{2} c^2 \xi \quad . \tag{16}$$

Taking into account the main aim of the investigation, equations (9) — (16) form a system of equations, which is the nonlinear mathematical model of the investigated easy pipeline system. For getting sensitivity coefficient matrix, these equations should be linearized.

In case of the equation (9):

$$\delta \nu = \delta \mu - \delta \rho \quad . \tag{17}$$

In case of the equation (10):

$$\delta c = \delta \dot{V} - 2 \delta d \quad . \tag{18}$$

In case of the equation (11):

$$\delta \operatorname{Re} = \delta c + \delta d - \delta \nu \quad . \tag{19}$$

In case of equations (12a) — (12d):

$$\delta \lambda = K \delta \operatorname{Re} \quad . \tag{20}$$

If $\operatorname{Re} < 2320$ then

$$K_a = -1 \, ; \tag{20a}$$

if $2320 < \operatorname{Re} < 8 \cdot 10^4$ then

$$K_b = -0{,}25 \quad ; \tag{20b}$$

if $2 \cdot 10^4 < \operatorname{Re} < 2 \cdot 10^6$ then

$$K_c = -\frac{0{,}1188}{0{,}0054 \operatorname{Re}^{-0{,}3} + 0{,}396} \quad ; \tag{20c}$$

if $10^5 < \operatorname{Re} < 10^8$ then

$$K_d = -\frac{0{,}074477}{0{,}0032 \operatorname{Re}^{-0{,}337} + 0{,}221} \quad . \tag{20d}$$

In case of the equation (13):

$$\delta h'_{cs} = 2 \delta c + \delta l + \delta \lambda - \delta d \quad . \tag{21}$$

In case of the equation (14):

$$\delta \Delta p_{cs} = \delta \rho + 2 \delta c + \delta l - \delta d + \delta \lambda \quad . \tag{22}$$

In case of the equation (15):

$$\delta h'_{sz} = 2 \delta c + \delta \xi \quad . \tag{23}$$

In case of the equation (16):

$$\delta\Delta p_{sz} = \delta\rho + 2\delta c + \delta\xi \quad . \tag{24}$$

Introducing the vector of dependent parameters

$$\underline{x}^T = \begin{bmatrix} \mu & \rho & \dot{V} & d & l & \xi \end{bmatrix} \ , \tag{25}$$

and the vector of independent parameters

$$\underline{y}^T = \begin{bmatrix} \nu & c & Re & \lambda & h'_{cs} & \Delta p_{cs} & h'_{sz} & \Delta p_{sz} \end{bmatrix} \ , \tag{26}$$

their coefficient matrices are:

$$\underline{\underline{A}} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -K & 1 & 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & -2 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & -2 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & -2 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{27}$$

$$\underline{\underline{B}} = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{28}$$

Because the system is investigated in different Reynolds-number intervals — using equations (7), (20), (27) and (28) the sensitivity coefficient matrices were determined. In last two intervals, the coefficient K depends on Reynolds-number. They are calculated by given Reynolds-numbers — see equations (29c) and (29d).
If $\mathrm{Re} < 2320 \ \rightarrow \qquad K_a = -1$:

$$\underline{\underline{D}}_a = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -2 & 0 & 0 \\ -1 & 1 & 1 & -1 & 0 & 0 \\ 1 & -1 & -1 & 1 & 0 & 0 \\ 1 & -1 & 1 & -4 & 1 & 0 \\ 1 & 0 & 1 & -4 & 1 & 0 \\ 0 & 0 & 2 & -4 & 0 & 1 \\ 0 & 1 & 2 & -4 & 0 & 1 \end{bmatrix} \quad ; \qquad (29a)$$

if $2320 < \mathrm{Re} < 8 \cdot 10^4 \quad \rightarrow \qquad K_b = -0{,}25:$

$$\underline{\underline{D}}_b = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -2 & 0 & 0 \\ -1 & 1 & 1 & -1 & 0 & 0 \\ 0{,}25 & -0{,}25 & -0{,}25 & 0{,}25 & 0 & 0 \\ 0{,}25 & -0{,}25 & 1{,}75 & -4{,}75 & 1 & 0 \\ 0{,}25 & 0{,}75 & 1{,}75 & -4{,}75 & 1 & 0 \\ 0 & 0 & 2 & -4 & 0 & 1 \\ 0 & 1 & 2 & -4 & 0 & 1 \end{bmatrix} \qquad (29b)$$

if $2 \cdot 10^4 < \mathrm{Re} < 2 \cdot 10^6 \quad (\mathrm{Re} = 1 \cdot 10^6) \quad \rightarrow \qquad K_c = \text{-}0{,}2999351775:$

$$\underline{\underline{D}}_c = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -2 & 0 & 0 \\ -1 & 1 & 1 & -1 & 0 & 0 \\ 0{,}2999 & -0{,}2999 & -0{,}2999 & 0{,}2999 & 0 & 0 \\ 0{,}2999 & -0{,}2999 & 1{,}7001 & -4{,}7001 & 1 & 0 \\ 0{,}2999 & 0{,}7001 & 1{,}7001 & -4{,}7001 & 1 & 0 \\ 0 & 0 & 2 & -4 & 0 & 1 \\ 0 & 1 & 2 & -4 & 0 & 1 \end{bmatrix} \qquad (29c)$$

if $10^5 < \mathrm{Re} < 10^8 \qquad (\mathrm{Re} = 5 \cdot 10^6) \quad \rightarrow \qquad K_d = \text{-}0{,}3369730350:$

$$\underline{\underline{D}}_d = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -2 & 0 & 0 \\ -1 & 1 & 1 & -1 & 0 & 0 \\ 0{,}337 & -0{,}337 & -0{,}337 & 0{,}337 & 0 & 0 \\ 0{,}337 & -0{,}337 & 1{,}663 & -4{,}663 & 1 & 0 \\ 0{,}337 & 0{,}663 & 1{,}663 & -4{,}663 & 1 & 0 \\ 0 & 0 & 2 & -4 & 0 & 1 \\ 0 & 1 & 2 & -4 & 0 & 1 \end{bmatrix} \qquad (29d)$$

4. The result of the sensitivity analysis

Knowing the sensitivity coefficient matrix $\mathbf{D}$, sensitivity of the system can be investigated by modification of independent variables vector $\delta\mathbf{x}$. Results of sensitivity analysis can be used for conclusions to come about features of the given system and its behavior in case of simulated failures or parameter uncertainty (for example instability of geothermal water viscosity). It is important to mention that changes of independent variables cannot be higher than about 1 or 5 %, depending on the intensity of the original model nonlinearity.

In case of pipeline systems, the independent variables can be classified to three categories:

The mode of functioning values determines the work of the system at the investigated time.

The technical system data characterize system structure and geometrical and other system parameters. These data have manufacturing anomalies and they can change during the system operation too. In our study, the investigated technical data are:

- the internal diameter of the tube,
- pipe fitting loss coefficient.

The physical parameters define the quality of the fluid. They are very interesting in case of the geothermal pipeline system when water parameters (for example salinity) — thereby the required pump power — can change easily. These variables are:

- dynamic viscosity,
- fluid density.

4.1. The investigation of the mode of functioning values uncertainties' effects

Fig. 1. shows the effects of 1% increase of the volume flow rate in case of different Reynolds-number intervals. The diagram demonstrates that constant cross-section, volume flow rate can be increased only by greater flow velocity, which generates greater Reynolds-number. Therefore, pipe loss coefficients will decrease by different degrees. In case of the stable laminar flow ($\mathrm{Re} < 2320$), it is observable that the pipe loss coefficient is more sensitive than in other Reynolds-number intervals. The relative increase of losses of the pipe (by Reynolds-number domains) is equal. It is worth to note that, these parameters are less affective in case of the stable laminar flow ($\mathrm{Re} < 2320$). Losses of the pipe fitting have the greatest sensitivity; they depend on the volume flow rate fluctuation.

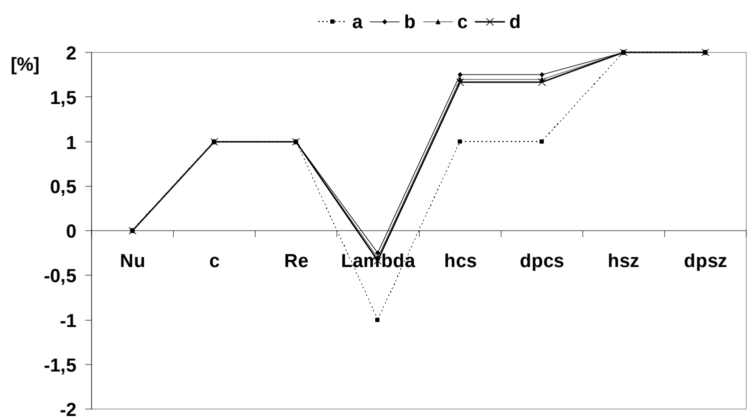

Fig.1. The sensitivity of the System Depending on the Volume Flow Rate ($\delta\dot{V} = +1\%$)

4.2. The investigation of technical system data uncertainties' effects

Fig. 2. shows the effects of 1% increase of the internal diameter of the tube. It can be seen that, diameter increase (which is cross section in Fig. 2.) adds up to the decreasing of the Reynolds-number. It can be noticed that, pipe loss coefficients will increase (in case of $\mathrm{Re} < 2320$ by most large measure) and tube losses will decrease by different degrees.

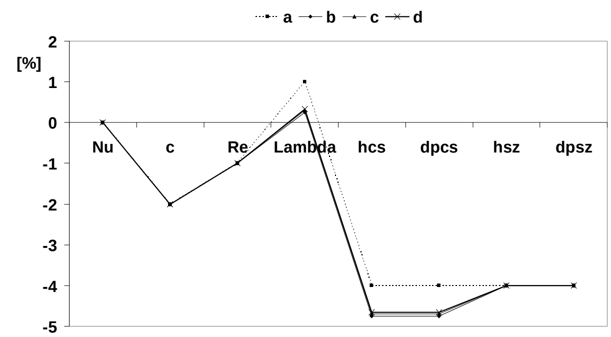

Fig. 2. The Sensitivity of the System Depending on the Internal Diameter ($\delta d = +1\%$)

Results of 1% increase of the pipe fitting loss coefficient can be seen in Fig. 3. The diagram shows, that the uncertainty of the pipe fitting loss coefficient generates only losses of the pipe fitting.

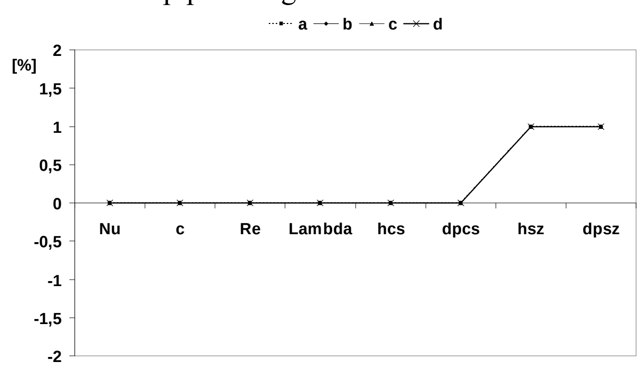

Fig. 3. The Sensitivity of the System, Depending on the Pipe Fitting Loss Coefficient ($\delta\xi = +1\%$)

4.3. The investigation of physical parameters of fluid uncertainties effects

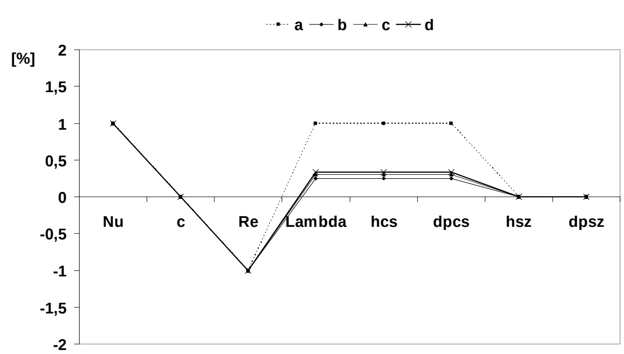

Fig. 4. The Sensitivity of the System Depending on the Dynamical Viscosity ($\delta\mu = +1\%$)

The effects of 1% dynamical viscosity increase are shown in Fig. 4. The diagram shows that dynamic viscosity increasing (in case of the constant fluid density) accrues kinematical viscosity and decreases the Reynolds-number. In this case, the pipe loss coefficient and losses of the pipe have similar sensitivities in each Reynolds-number interval. It is perceptible, that the uncertainty of the dynamical viscosity has not effect on pipe fitting losses.

It can be established in Fig. 5, that effects of the fluid density are in contradiction with results of the dynamic viscosity anomaly. In $\mathrm{Re} < 2320$ Reynolds-number interval, the pipe loss coefficient and head loss of the pipe have the greatest sensitivity.

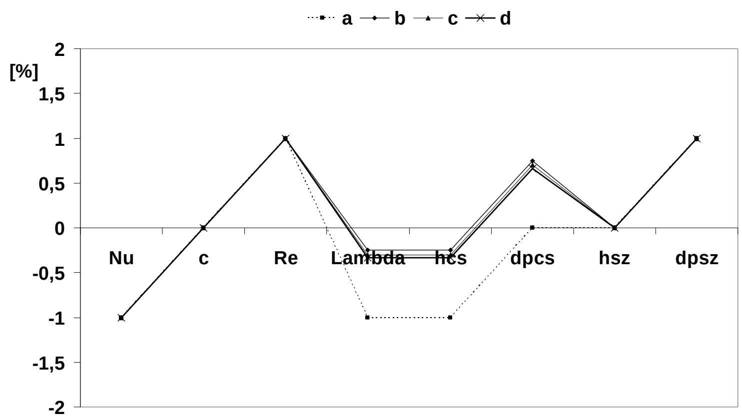

Fig. 5.The Sensitivity of the System Depending on the Fluid Density ($\delta\rho = +1\%$)

During the investigation of physical fluid parameters' effects, the several-parameter sensitivity analysis is worth to be performed. It is probable that the change of the feature (for example salinity) of the fluid can produce the change of its density and dynamical viscosity. Therefore, $\delta\mu = +1\%$ and $\delta\rho = +1\%$ situation will be investigated as the modeling of the increase of the salinity. Its results are shown in Fig. 6. The graph shows that, the increase of the water salinity has not influence on system parameters excluding the pressure losses.

The statement mentioned above is misleading. Because the density and dynamical viscosity of the fluid will not change equally: they depend on water salinity. In case of 1% water salinity increase, the density increases with 0,21 % and the dynamical viscosity of the fluid increases with 4,885 % [4]. Therefore the vector of the relative change of independent variables will modified to:

$$\delta \underline{x}^T = \begin{bmatrix} 4{,}885 & 0{,}21 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad (30)$$

Results of the modeling are shown in Fig. 7.

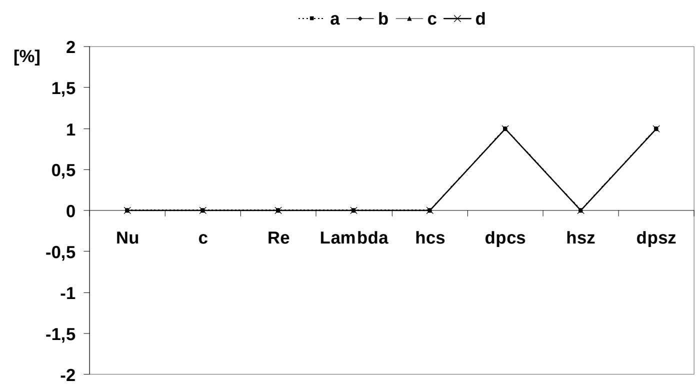

Fig. 6. The Sensitivity of the System ($\delta\mu = +1\%$ and $\delta\rho = +1\%$)

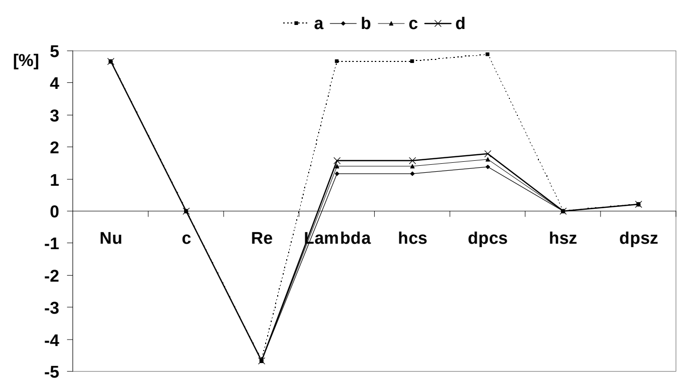

Fig. 7. The Sensitivity of the System ($\delta\mu = +4{,}885\%$ and $\delta\rho = +0{,}21\%$)

The first conspicuous conclusion is: the dependent system variables have the highest sensitivities depending on the salinity of the water. Correspondingly with the result of the one parameter sensitivity analysis, the system is most sensible in case of the stable laminar flow ($\mathrm{Re} < 2320$).

5. Conclusions, future works

The author of this paper would like to point out the importance and possibilities of use of the mathematical model uncertainty analysis. This basically theoretical paper has shown the sensitivity analysis. Then the methodology of sensitivity test, which is based on uncertainty analysis, has been shown by the case study of an easy pipeline system.

During prospective scientific research related to this field of applied mathematics and technical system modeling, the author would like to complete following tasks:

– the sensitivity analysis of complex pipeline system and pipe-network;

– data collection for depicting correctly the influence of the salinity to water physical parameters and required pump power;
– to investigate possibilities of the adaptation of linear interval equations for parametric pipeline system uncertainty analysis.

R E F E R E N C E S

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