METAL SHEET PLASTIC ANISOTROPY IDENTIFIED BY A MIXED NUMERICAL-EXPERIMENTAL METHOD

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Comportarea plastică a tablelor metalice anizotrope este pusă în evidență combinând rezultatele încercării uniaxiale la tracțiune cu încercările eteroogene biaxiale la tracțiune pe probe cruciforme. Încercările uniaxiale au permis punerea în evidență a ecruisajului și a unor parametri ai curgerii legați de deformarea plană anizotropă și de tensiunea anizotropă. Rezultatele obținute în încercările biaxiale au fost analizate folosind o tehnică de optimizare care constă din fitarea numerică a câmpurilor de deformare cu cele experimentale. Prin acest procedeu au fost obținuți parametrii de material care descriu forma suprafeței de curgere. În final au fost efectuate simulări ale încercării de ambutisare pentru a verifica validitatea caracterizării.

The plastic behavior of anisotropic sheet metals is identified by combining the results of classical uniaxial tensile tests and of heterogeneous biaxial tensile tests on cruciform specimens. The uniaxial tests allow the identification of strain-hardening and of some parameters of the yield function relating to planar strain anisotropy and stress anisotropy. The results obtained in the biaxial tests are analyzed using an optimization technique that consists of fitting numerical strain fields with experimental ones. The material parameters describing the shape of the yield surface are obtained by this procedure. Finally, simulations of the cup drawing test are performed to check the validity of the identification.

Key words: cold worked metal sheets, strain hardening, anisotropy, numerical experimental identification

1. Introduction

The properties and behaviour of cold worked metallic products are severely affected by strain hardening and anisotropy. The identification of metal sheet plastic anisotropy is usually performed using a series of loading tests under various linear stress-states, i.e., uniaxial tension tests, plane-strain tension tests on

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wide specimens, simple shear tests and/or hydraulic bulge tests [1]. Instead of using this classical method, the determination of anisotropic yield surfaces is approached in this paper by means of a mixed numerical-experimental method calling upon the analysis of heterogeneous strain fields obtained on complex cruciform specimens tested in biaxial tension. The material parameters are identified in two steps: first, uniaxial tensile tests are performed to characterize strain-hardening and some parameters of the yield function accounting for the variations of the anisotropy coefficient $R$ and of the uniaxial yield stress $\sigma$ as a function of the angle $\alpha$ between the rolling direction and the tensile axis; second, the material parameters relating to the shape of the yield surface in the expansion range are identified by combining experimental strain fields obtained in biaxial tensile tests, finite element simulations and an optimization technique.

2. Experimental and numerical procedures

The strain analysis was carried out in the uniaxial and biaxial tensile tests. The strain measurement technique is based on a digital image correlation method developed by Brunet et al. [2]. Each image of the deformed specimen is compared with the image of the undeformed surface using a grey level correlation coefficient. In order to perform more easily image correlation, a random grey level is needed on the samples before straining (Fig. 1). This is obtained using two sprays of painting, consisting of a black speckle applied on a uniform white background.

![Fig. 1. Speckle aspect (a) at the beginning and (b) at the end of a biaxial tensile test](image)

The correlation technique is based on the search of a displacement field for a sub-image, or pattern. Correlation technique uses contrast between pixels of the same pattern. The in-plane displacement field of each pattern is then used to
calculate the strain field. The software (trade name Icasoft) allows us to obtain maps of the Green-Lagrange and/or Hencky strain measures.

Two types of biaxial specimens were designed, with the aim of offering a high sensitivity of the strain field obtained under biaxial stretching to plastic anisotropy. The stress range goes from uniaxial tension (UT) to plane-strain tension (PST) for the first type of specimen (UT/PST specimen, Fig. 3a), and from uniaxial tension to equibiaxial tension (EBT) for the second type (UT/EBT specimen, Fig. 3b).

The finite element simulations were performed with the Abaqus/Explicit code, where the anisotropic yield model proposed by Ferron et al. [4] (FMM model) is implemented. The plane-stress version of the criterion is based on a polar-coordinate representation of yield surfaces in principal stress plane \((\sigma_1, \sigma_2)\). In this representation, a point on the yield surface is defined by the function \(g(\theta, \alpha)\), where \(g\) is the radius to a point of the yield surface, \(\theta\) is the polar angle on the yield surface, and \(\alpha\) is the angle between the orthotropic axes \((x_1, x_2)\) and
the principal stress axes \((s_1, s_2)\). The principle of this description is presented on Fig. 4, where the yield surfaces are normalized by the effective stress, and are drawn for different values of the angle \(\alpha\). The function \(g(\theta)\) for transversely isotropic (normally anisotropic) materials is defined by:

\[
(1-k)g(\theta)^6 = (\cos^2 \theta + A \sin^2 \theta)^3 - k \cos^2 \theta (\cos^2 \theta - B \sin^2 \theta)^2
\]  

(1)

and its extension to transverse anisotropy is given by:

\[
g(\theta, \alpha)^w = g(\theta)^w - 2 \sin \theta \cos^2 \theta \cos 2\alpha + b \sin^2 \theta \cos^2 \alpha
\]  

(2)

The Von Mises and Drucker’s isotropic yield criteria can be retrieved as particular cases of (1), as well as Hill’s quadratic yield criterion as a particular case of (2).

Fig. 4. Sketch of the yield surface in principal stress space \((\sigma_1, \sigma_2)\). The yield loci are parameterized by the angle \(\alpha\) between the rolling direction and the principal stress axis \(s_1\). The yield function is used with the assumption of isotropic hardening. Strain-hardening is described by the Swift law:

\[
\sigma = K (\varepsilon^p + \varepsilon_0)^N
\]

where \(\sigma\) is the effective stress, \(\varepsilon^p\) is the effective plastic strain, and \(K, \varepsilon_0\) and \(N\) are material constants.
Fig. 5. Influence of yield surface shape on strain distributions for UT/PST specimens.

The sensitivity of strain fields to material anisotropy is illustrated on Fig. 5, for UT/PST specimens with the same geometry (same radius of the central hole). The simulations are made for transversely isotropic materials with the same value of the anisotropy coefficient, $R = 2$, yet with two different yield surfaces, Fig. 5a. The thickness strain is larger in the heads of the specimen with Hill’s model, Fig. 5b, while it is most intense in the connecting parts between the heads with the FMM model, Fig. 5c. This is explained by the larger ratio of plane-strain yield stress $\sigma_p$ to uniaxial yield stress $\sigma_u$ with Hill’s criterion.

The procedure defined to identify the material parameters of the yield function is as follows:

**Parameters defining the yield surface:**

- 8 parameters: $k, A, B, m, n, p, a, b$

**First step** : analysis of uniaxial tensile tests:

**Data** : $R_0$, $R_{45}$, $R_{90}$

**Procedure** : take $k = 0$, determine the $(m, n, p)$ values fitting the evolution of $\sigma(\alpha)$

- (the parameters $A$, $a$ and $b$ are determined knowing $R_0$, $R_{45}$ and $R_{90}$)

- keep the $(m, n, p)$ values for the continuation of identification.

**Second step** : analysis of heterogeneous biaxial tensile tests:

**Data** : $R_0$, $R_{45}$, $R_{90}$, experimental strain fields

**Procedure** : use an iterative procedure to determine the $(k, B)$ pair minimizing

- the difference between experimental and calculated strain fields

- (for each $(k, B)$ pair, the parameters $A$, $a$ and $b$ are determined knowing $R_0$, $R_{45}$ and $R_{90}$)
3. Results and Discussion

The results are presented for a mild steel of deep-drawing quality supplied by ArcelorMittal.

**Uniaxial tensile tests**

The effective stress-strain curves in uniaxial tension along the rolling direction (RD), transverse direction (TD) and diagonal direction (DD) are shown on Fig. 6. The experimental curves are very well described by the Swift law. The average values of material constants are: \( K = 510 \text{ MPa}, \ \varepsilon_0 = 0.004 \) and \( N = 0.2366 \).

![Fig. 6. Experimental stress-strain curves fitted by the Swift hardening law.](image_url)

The anisotropy coefficient \( R \), measured by means of the image correlation analysis, and the flow stress \( \sigma \) (normalized by the flow stress along the RD) are shown in Fig. 7 for the three testing directions. A very good fit is obtained for both \( R(\alpha) \) and \( \sigma(\alpha) \) with the FMM model, with the values: \( m = 2, \ n = 3 \) and \( p = 2 \).

![Fig. 7. Angular variations of the anisotropy coefficient \( R \) and of the flow stress \( \sigma \).](image_url)

**Biaxial tensile tests**

The strain fields used for the identification procedure are chosen to be the major principal strains \( \varepsilon_1 \) along the following lines:
- the symmetry axes of UT/PST specimens corresponding to the RD and the TD,
- the line inclined at 45° from the preceding axes (diagonal direction, DD),
- the symmetry axes of UT/EBT specimens corresponding to the RD and the TD.
The finite element simulations are run by changing the \((k, B)\) pair so as to minimize the cost function:

\[
\Phi(P) = \frac{1}{n} \sum_{i=1}^{n} \left( \frac{\varepsilon_i(k, B) - \varepsilon_i^{\text{exp}}}{\varepsilon_i^{\text{exp}}} \right)^2
\]

where \(\varepsilon_i(k, B)\) is the value of the major principal strain \(\varepsilon_1\) calculated at point \(i\) with the current \((k, B)\) pair and \(\varepsilon_i^{\text{exp}}\) is the experimental value at point \(i\). The best fit is obtained for this material with \((k = -1, B = 0)\). The parameters of the yield function are given in Table 1. The yield surfaces are presented on Fig. 8, for the two orientations of principal stress axes at 0° and 45° from the RD.

### Table 1. Best-fit parameters of the yield function (2).

<table>
<thead>
<tr>
<th>Anisotropy parameters</th>
<th>(k)</th>
<th>(A)</th>
<th>(B)</th>
<th>(m)</th>
<th>(n)</th>
<th>(p)</th>
<th>(a)</th>
<th>(b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FMM model</td>
<td>-1</td>
<td>4.62</td>
<td>0</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>-0.099</td>
<td>0.74</td>
</tr>
</tbody>
</table>

![Fig. 8. Yield surfaces determined for steel. Comparison with Hill’s quadratic yield criterion.](image)

**Cup drawing tests**

The cup drawing test was selected to check the ability of the yield function to provide a good description of the formation of ears. A specific device was used to this end. The punch diameter is equal to 30 mm and the circular blanks have a diameter of 66 mm, giving a drawing ratio of 2.2. The predicted and measured earing profiles in the cup drawing tests are shown in Fig. 9. It can be observed that the height of ears predicted with the FMM criterion is in very good agreement with experiments. This is to be compared with the in-plane variations obtained for \(\sigma(\alpha)\) (see Fig. 7).
4. Conclusions

The identification of the anisotropic plastic behavior of a steel has been performed by combining the results of uniaxial tensile tests and heterogeneous biaxial tensile tests. The obtained material parameters give a very good description of the anisotropic behavior observed in the cup drawing test.

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